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Abstract

In this paper we show that in a pure exchange economy it is possible to improve the social welfare along an efficient path. This path will be called the Negishi map. Moving the relative weights of the agent in a social welfare utility function, we obtain an efficient path of allocations and social weights, such that along this path the social welfare level change. Moving along this path it is possible to reach a maximum social welfare. The efficient allocation maximizing the social welfare is characterized by the fact that the individual utilities have the same value. This level will be called the Negishi number of the economy. Such allocation is not necessarily an allocation corresponding to a walrasian equilibrium so, the participation of a benevolent policy maker can have sense. We introduce a definition of developed economy. Finally, the relations between changes in utilities and changes in social weights is analized.
Keywords: *Negishi approach, Negishi map, social welfare.*

JEL code: D6; D51.

Resumen

En este trabajo mostramos que una econmía de intercambio puro alcanza su máximo nivel de bienestar siguiendo una trayectoria eficiente a lo largo de un camino diferenciable al que llamamos camino de Negishi. El máximo nivel posible alcanzable por una economía es un punto

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en esta trayectoria, que permite definir el llamado número de Negishi. Este valor no depende de la distribución de las dotaciones iniciales, sólo de su agregado. La asignación de recursos que corresponde a este número es de alguna forma *igualitaria* en cuanto al nivel de felicidad que cada individuo alcanza. La posibilidad de alcanzar este nivel de felicidad en forma descentralizada, depende de la distribución de las dotaciones iniciales y no de su agregado. Esto nos lleva a definir como economías desarrolladas aquellas en la que este máximo nivel de bienestar social es alcanzable en forma descentralizada, es decir, como valor social correspondiente a una asignación de recursos correspondiente a un equilibrio walrasiano. Finalmente mostramos una relación inversa entre los posibles cambios de niveles de bienestar para una economía y las características de la llamadas funciones exceso de utilidad de cada agente.
Palabras claves: *Teoría de Negishi, mapa de Negishi, bienestar social*
Calsificación JEL : D6; D51,

1 Introduction

In this paper we introduce a relation between efficiency, equilibrium and social welfare. This relation is setting from the Negishi approach that consists in obtaining the Pareto optimal allocation by maximizing a social welfare function. In the classical literature on general equilibrium the endowments are generally fixed and there is not an exact definition of the concept of social optimal allocation. In this paper we consider the possibility to move the initial resources and we introduce a criterium to measure the social welfare of a Pareto optimal allocation. As it is well known, the social welfare associated with a no Pareto optimal feasible allocation can be improved by means an efficient allocation. So the social maximum welfare level if there exists, it can be only reached in a Pareto optimal allocation. Efficiency implies allocations that leads to utility vectors to the Pareto frontier (see for instance [Mas-Colell, A. (1975)]). We explore the relation between social weights, initial resources, and social welfare. We introduce the definition of Negishi path, and we show that this path does not depend on the distributions of the initial endowments. This means that this path, is the same for all economy with utilities and total resources fixed. Assuming that the social value of an allocation x is given by a social utility function given by $\sum_{i=1}^{n} \lambda_i u_i(x_i)$, where $u_i, i = 1, ..., n$ are the utility functions of the agents of the economy, we explore the relation between social weights, initial resources, social welfare and we analyze the main characteristics of the allocation that maximize the social welfare. We show that for this allocation, the individual level of welfare is the same for each agent.

We analyze the possibility that this maximum level of social welfare, can be reached without the participation of a central planner. Competitive economies can reach only equilibrium allocation by their owns forces. Equilibrium allocations are Pareto optimal allocations, but the possibility that an efficient allocation is a walrasian equilibrium depends, not only in the amount of total resources (as in the case of the Pareto optimal allocations), but also in the initial distribution of these resources.

Using the Negishi approach, we show that each Pareto optimal allocation has associated a vector $\lambda = (\lambda_1, ..., \lambda_n)$, that plays the role of the relative weights of the agents in the aggregate social utility function given by $\sum_{i=1}^{n} \lambda_i u_i$. By means of this function we obtain a measure of the social welfare associate with each Pareto optimal allocation. Each coordinate of this vector can be interpreted as a relative measure of the real weights of the agents in the market. As it is well known, associate with each economy there exists only a subset of Pareto optimal allocations that can be walrasian allocations, and there is only one distribution of social weights associate with each Pareto optimal allocation in this subset. These distributions of social weights will be called social equilibria. So, the economy can reach only the social welfare levels associate with these set of social weights and allocations. So, the possible levels of social welfare reachable in a decentralized way for this economy, depends on the initial distribution of the resources. The maximum level of social welfare is reachable if and only if the initial distribution of resource allows that the Pareto optimal allocation corresponding with this level of welfare is a walrasian allocation.

Finally we analyze the relation between changes in utilities of the different agents of the economy and changes in social weights. We try to recover the main characteristics of the economy analyzing the set of social equilibria.

2 Pareto optimallity and social welfare

Consider a pure exchange economy with n agents and l goods. The consumption space $X \subseteq R^l_+$ is the same for each agent. Agents $(i = 1, 2, ..., n)$ has quasi-concave utility functions u_i, and endowments $w_i \in R^l_+$. Total resources $\Omega = \sum_{i=1}^n w_i$ are fixed. We recall that an allocation $x = (x_1, ..., x_n) \in X^n$ is feasible if and only if $\sum_{i=1}^n x_i = \sum_{i=1}^n w_i$. Let us denote by $\mathcal{F}$ the set of feasible allocations. The set

$$U = \{(u_1, u_2, ..., u_n) \in R^n : \text{ there is a } x \in \mathcal{F} \text{such that } u_i \leq u_i(x_i), i = 1, ..., n.\}$$

is called the *utility possibility set*. The Pareto frontier of this set is denoted by

$$UP = \left\{(u_1, u_2, ..., u_n) \in U : \nexists (u'_1, u'_2, ..., u'_n) \in U \text{ with } u'_i \geq u_i \ \forall i \text{ and } u'_i > u_i \text{ for some } \ i\right\}.$$

Let $U(x) \in R^n$, be the vector representing the level of utility attained by each agent given the allocation $x \in X^n$; i.e., $U(x) = (u_1(x), u_2(x), ..., u_n(x))$. If u and u' are vectors in R^n we denote $u' \geq u$ if and only if $u'_i \geq u_i, \forall i \in \{1, 2, ..., n\}$.

Note that a first criterion to maximize the social welfare is to choose a Pareto optimal allocation. In the following proposition we give the definition of Pareto optimal allocation in terms of the utility possibility set.

Proposition 1 *A feasible allocations x is Pareto optimal if and only if $U(x) \in UP$*

Proof: Let x be a feasible allocation. If $U(x) \notin UP$ then there exists $u' \in U$ with $u' \neq U(x)$ and $u' \geq U(x)$. But $u' \in U$ and then there exists a feasible allocation y such that $U(y) \geq U(x)$, implying that $U(y) \geq U(x)$ and $U(y) \neq U(x)$. Then x is not Pareto optimal.

Reciprocally, if x is not a Pareto optimum then there exists a feasible allocation y such that $u_i(y_i) \geq u_i(x_i)$, for all i with strict

Suppose now that the distributional principles of the society are summarized by the social welfare function:

$$W_\lambda(x) = \sum_{i=1}^{n} \lambda_i u_i(x_i), \tag{1}$$

where $x \in X^n$ is a feasible allocation and $\lambda = (\lambda_1, ..., \lambda_n)$ with $\lambda_i \geq 0, \forall i = 1, ..., n$, is the given vector of social weights. Let

$$\Delta = \left\{ \lambda \in R^n : \sum_{i=1}^{n} \lambda_i = 1 \text{ with } 0 \leq \lambda_i \leq 1, \forall i \in \{1, ..., n\} \right\} \tag{2}$$

be the simplex and Δ_+ be the relative interior of the simplex.

As it is well known, if $x^* = (x_1^*, ..., x_n^*)$ with $x_i^* \in X$ solves the social welfare maximization problem given by

$$\max_{x \in \mathcal{F}} W_\lambda(x) \text{ with } \lambda \in \Delta_+ \tag{3}$$

them x^* is a Pareto optimal allocation and $(u_1(x_1^*), ..., u_n(x_n^*) \in UP$. Reciprocally, for each Pareto optimal allocation x^* there exists $\lambda^* \in \Delta_+$ such that x^* solves $\max_x W_{\lambda^*}(x) \;\; \forall x \in \mathcal{F}$.

Let PO be the set of Pareto optimal allocations and $x^* : \Delta_+ \to PO$ be the map that assign to each $\lambda \in \Delta_+$ the solutions of the maximization problem (3). Note that this map is well defined under the hypothesis of strictly-concavity of the utility functions.

Let $\mathcal{E} = \{X, u_i, w_i, I\}$ be a pure exchange, where $u_i : X \to R$ are the utility functions, w_i the endowments, for all $i \in I$ where I is a finite set of index, one for each consumer. Let us now introduce the following definition:

Definition 1 *Let $\mathcal{E}$ be an economy, we define the Negishi path of the economy $\mathcal{E}$, as the application $\mathcal{C}_N : \Delta_+ \to \Delta_+ \times \mathcal{PO}$ defined by*

$$\mathcal{C}_N(\lambda) = \{(\lambda, x^*(\lambda)) \;\; \forall \; \lambda \in \Delta_+\}.$$

Note that the projection $\Pi : \Delta \times X^n \to X^n$ restricted to $\mathcal{C}_N$ $\pi = P_{\mathcal{C}_N} \to X^n$ verifies that its image: $IM[\pi] = Im[P_{\mathcal{C}_N}] = \mathcal{PO}$. Then $U(\pi(\lambda, x^*(\lambda)) \in UP$.

Definition 2 *Given an economy $\mathcal{E} = \{X, u_i, w_i, I\}$ we say that the economy $\mathcal{E}' = \{X, u_i, w_i', I\}$ is obtained by a redistribution of the initial resources from the economy $\mathcal{E}$ if and only if $w' \neq w$ and $\sum_{i=1}^{n} w_i' = \sum_{i=1}^{n} w_i$.*

The following theorem summarize the main properties of the Negishi path:

Theorem 1 *Let $\mathcal{E}$ be an economy with strictly-concave utility functions, then:*

1. The Negishi path is C^1, this means that, $\frac{\partial}{\partial \lambda_i}\mathcal{C}_N(\lambda)$ is continuous for al $i = 1, 2, ..n$.

2. The Negishi path is the same for all economy obtained by a redistribution of the initial resources from the economy $\mathcal{E}$.

Proof: It is easy to see that from de implicit function theorem applied to the first order conditions that define $x^*(\lambda)$ it follow that the map $x^* : \Delta_+ \to \mathcal{PO}$ is differentiable with continuous derivatives. To prove (2) note that the set of feasible Pareto optimal allocations of an economy, does not depend on the distribution of its own resources, this map can change only if utilities or total resources change. Then the map $x^* : \Delta_+ \to \mathcal{PO}$ is the same for all economy obtained by a redistribution of the initial resources from the economy $\mathcal{E}$.[$\cdot$]

In the next section we will solve the maximization problem in the case of a one period pure exchange economy where total resources are fixed.

3 Pareto optimality and welfare in a pure exchange economy with fixed resources

Let $\mathcal{E} = \{X, u_i, w_i, i = 1, 2, ..n\}$ be a pure exchange economy where total resources Ω are fixed and let $w = (w_1, ..., w_n) \in X^n$ be the vector of initial endowments. We denote by $u = (u_1, ..., u_n) \in R^n$ the profile of utility functions, and by $w = (w_1, ..., w_n) \in X^n$ the vector of the initial endowments. We say that w' is a *reallocation* of w if $w' = (w'_1, ..., w'_n) \neq w = (w_1, ..., w_n)$ and $\sum_{i=1}^n w_i = \sum_{i=1}^n w'_i$.

We will show that if social weights are fixed, then all Pareto optimal allocations x^* have the same level of social welfare. Let consider the maximization problem:

$$\begin{aligned} &W_\lambda(x) = max_{x \in X^n} \textstyle\sum_{i=1}^n \lambda_i u_i(x_i) \\ &s.t.\ \textstyle\sum_{i=1}^n x_i = \sum_{i=1}^n w_i = \Omega. \end{aligned} \tag{4}$$

Note that the solution of the problem does not depend on the initial distribution of resources w, but it depends on the total resources $\sum_{i=1}^n w_i = \Omega$. So the Negishi map for these two economies is the same. This is the case of the model of international emission of CO_2 trading described in [Burguet, R.; Sempere, J.], where all initial allocations of permission for emissions are reallocations, so for a given distribution of social weights, $\lambda \in \Delta_+$ all Pareto efficient emission associate with each reallocation have the same level of social welfare. Then if we hope to reduce the quantity of emissions we need to change the distribution of social weights.

Suppose now that the central planner is able to choose the social weights. From the set of solutions $x^*(\lambda)$ of problem (4) he prefers to choose λ^* such that $W_{\lambda^*}(x^*(\lambda^*)) \geq W_\lambda(x^*(\lambda))$ for all $\lambda \in \Delta$. Following the Fenchel theorem, this path can be obtained by means of the following minimization program, (the dual of problem (4))

$$\begin{array}{l} \min_{\lambda} W(\lambda, x^*(\lambda)) \\ s.t.\ \lambda \in \Delta_+, \end{array} \tag{5}$$

where $W(\lambda, x^*(\lambda)) = \sum_{i=1}^n \lambda_i u_i(x_i^*(\lambda))$.

Theorem 2 *There exists a solution $\lambda^* \in \Delta$ to the problem (5). This solution verifies that the utilities of the corresponding allocations of resources $u_i(x_i^*(\lambda^*))$ of all the consumers are equal.*

Proof: Note that $W(\lambda, x^*(\lambda))$ is a convex function of λ. To see this, suppose that the allocations $\bar{x}$ and $\bar{\bar{x}}$ are supported by $\bar{\lambda}$ and $\bar{\bar{\lambda}}$, respectively. And let x^c the Pareto optimal allocation supported by, $\lambda^c = \alpha\bar{\lambda} + (1-\alpha)\bar{\bar{\lambda}}$; $0 \leq \alpha \leq 1$. Therefore $W(\lambda^c, x^*(\lambda^c)) \leq \alpha \sum_{i=1}^n \bar{\lambda}_i u_i(x_i^*(\bar{\lambda})) + (1-\alpha)\sum_{i=1}^n \bar{\bar{\lambda}}_i u_i x_i^*(\bar{\bar{\lambda}})) = \alpha W(\bar{\lambda}, x^*(\bar{\lambda})) + (1-\alpha) W(\bar{\bar{\lambda}}, x^*(\bar{\bar{\lambda}}))$.

From the first order conditions of problem (4) it follows that

$$\lambda_i grad\ u_i(x_i) = \gamma;\ i = 1, ..., n \tag{6}$$

where $\gamma(\lambda) = (\gamma_1(\lambda), ..., \gamma_l(\lambda))$ is the set of Lagrange multipliers and $grad\ f(x)$ is the gradient of function f . From $\sum_{i=1}^n x_i(\lambda) = \Omega$ it follows that

$$\sum_{i=1}^n \frac{\partial x_i^*(\lambda)}{\partial \lambda_h} \equiv 0, \quad \forall h \in \{1, ..., n\} \tag{7}$$

where $\frac{\partial x_i}{\partial \lambda_h} = \left(\frac{\partial x_{i1}}{\partial \lambda_h}, ..., \frac{\partial x_{il}}{\partial \lambda_h}\right)$. Now substituting $\lambda_n = 1 - \sum_{i=1}^{n-1} \lambda_i$ in $x^*(\lambda)$ and taking derivatives with respect to λ_k for $k = 1, ..., (n-1)$ we obtain:

$$\frac{d}{d\lambda_k} W(\lambda, x^*(\lambda)) = \sum_{i=1}^n \left\{ \left[\lambda_i \frac{\partial u_i(x_i^*(\lambda))}{\partial x_{i1}^*} \sum_{h=1}^l \frac{\partial x_{i1}^*(\lambda)}{\partial \lambda_h} \frac{d\lambda_h}{d\lambda_k} \right] + ... \right.$$

$$\left. ... + \left[\lambda_i \frac{\partial u_i(x_i^*(\lambda))}{\partial x_{il}^*} \sum_{h=1}^l \frac{\partial x_{il}^*(\lambda)}{\partial \lambda_h} \frac{d\lambda_h}{d\lambda_k} \right] \right\} + u_k(x_k^*(\lambda)) + u_n(x_n^*(\lambda)) \frac{d\lambda_n}{d\lambda_k} = 0, \quad k = 1, 2, ..., n-1 \tag{8}$$

where

$$\frac{d\lambda_h}{d\lambda_k} = \begin{cases} 1 & \text{if } h = k \\ -1 & \text{if } h = n \\ 0 & \text{elsewhere} \end{cases} \tag{9}$$

for all $k = 1, ..., n$. Substituting (6) in (8) we obtain:

$$\frac{d}{d\lambda_k} W(\lambda, x^*(\lambda)) = \sum_{i=1}^n \sum_{j=1}^l \gamma_j(\lambda) \left[\sum_{h=1}^l \frac{\partial x_{ij}^*}{\partial \lambda_h} \frac{\partial \lambda_h}{\partial \lambda_k} \right] + u_k(x_k(\lambda)) - u_n(x_n(\lambda)) = 0,\ \forall k \neq n. \tag{10}$$

Using the equalities obtained in (9) it follows that

$$\sum_{i=1}^{n}\sum_{j=1}^{l}\gamma_j(\lambda)\left(\frac{\partial x_{ij}^*(\lambda)}{\partial\lambda_h}-\frac{\partial x_{ij}^*(\lambda)}{\partial\lambda_n}\right)+u_k(x_k^*(\lambda))-u_n(x_n^*(\lambda))=0,\ \ k=1,...,(n-1). \qquad (11)$$

Finally, using (7) we obtain that:

$$\frac{d}{d\lambda_k}W(\lambda,x^*(\lambda))=u_k(x_k^*(\lambda))-u_n(x_n^*(\lambda))=0,\quad k=1,..,(n-1). \qquad (12)$$

Then we have that social welfare is maximized when λ^* verifies:

$$u_1(x_1^*(\lambda^*))=...=u_n(x_n^*(\lambda^*)) \qquad (13)$$

implying that the allocation maximizing the welfare is a form of *egalitarian solution.*[$\cdot$]

Associate with this allocation there is a distribution of social weights λ^* and a corresponding level of social welfare W^* such that $W^*=W(\lambda^*,x^*(\lambda^*))\geq W_\lambda(x^*(\lambda))\ \forall\lambda\in\Delta_+$ this level of welfare will be called the Negishi number of the economy.

Definition 3 *The number $W^*=W(\lambda^*,x^*(\lambda^*))$ will be called the Negishi number of the economy $\mathcal{E}$ and it will be denoted by $\mathcal{N}_N$. The point $(\lambda^*,x^*(\lambda^*))\in\mathcal{C}_N$ solving the maximization problem (5) will be denoted by ms.*

From the second welfare theorem we know that the central planner can implement the allocation $x^*(\lambda^*)$ as the corresponding allocation of a walrasian equilibrium, but the necessary information to do it is too large. But, certainly the second welfare theorem, is a useful theoretical result that is far from being a prescription for real policy. On the other hand, observe that the Negishi number of the economy, depends on the utilities representing the preferences of the agents, however the fact that these are the characteristic of the allocation ms with de maximum level of welfare, is independent of the individual utilities. The next point is to analyze the existing relations between allocations of equilibrium and those that assure a maximum level of social welfare.

4 Social welfare, reallocation of endowments and equilibria

Consider an economy $\mathcal{E}$ with utility functions given by $u_i, i=1,...,n$ and endowments $w=(w_1,...,w_n)$. Total resources are symbolized by $\Omega=(\Omega_1,...,\Omega_n),\ \Omega_i>0, i=1,..,n$. In this section we will analyze the relationship between social weights, social welfare, and walrasian equilibria.

A distribution of social weights λ is a social equilibrium if and only if it is a zero of the excess utility function, i.e. if and only if the point $(\lambda, x(\lambda^*) \in \mathcal{C}_N$ verifies

$$e(\lambda, w) = (e_1(\lambda, w), ..., e_n(\lambda, w)) = 0, \tag{14}$$

where

$$e_i(\lambda, w) = grad[u_i(x_i^*(\lambda)](x_i^*(\lambda) - w_i), \forall i = 1, 2, ..., n \tag{15}$$

The possibility of multiple social equilibria implies the possibility that equal economies in their fundamentals have different performances, and levels of social welfare. Conditions for uniqueness of this equilibria are given in ([Accinelli, E. (1996)]). It is not hard to be convinced that each allocation of resources $x^*(\lambda)$ solving (4), and such that $e(\lambda, w) = 0$ is an allocation corresponding to a walrasian equilibrium. And reciprocally, for each walrasian allocation x^* there exists the corresponding λ such that $x^* = x^*(\lambda)$ and $e(\lambda, w) = 0$.

On the other hand, given that the solution of the system of equations $e(\cdot, w) = 0$ depends on the initial distribution of resources, we will denote the allocation cooresponding to a solution of this problem by $\lambda(w)$. Note hat after a reallocation of resources it is possible that $e(\lambda(w), w') \neq 0$ and then the equality will be verified by a different $\lambda' = \lambda(w')$ associate with the allocation $x(\lambda(w')) \neq x(\lambda(w))$ and such that $e(\lambda', w') = 0$. The social welfare associated with the allocation $x(\lambda(w'))$ need not to be the same than the social welfare associated with the allocation $x(\lambda(w))$. Nevertheless $(\lambda(w), x^*(\lambda(w))$ and $(\lambda(w'), x^*(\lambda(w'))$ are points in $\mathcal{C}_N$.

So, our next question is the following: if for a given the distributions of initial resources w it is not possible to reach in a decentralized way the maximum social welfare corresponding to the Negishi number for the economy then, then can we find a mechanism such that the economy reach this level of social welfare? The answer is, if such rule exist is a mechanism to reallocate the initial resources of the economy.

Note that no necessarily the vector λ^* maximizing $W(\lambda, x^*(\lambda))$ define an allocation $x^*(\lambda^*)$ corresponding to a walrasian equilibrium. The possibility to reach this maximum level of social welfare for an equilibrium allocation depends on the distribution of the initial resources. If this is not the case, then the economy can not reach in a decentralized way, the Negishi number. Because, following its owns rules, a competitive economy can reach only a social welfare level associated with an equilibrium allocation. This conclusion gives rise the possibility to classify the economies in developed or underdeveloped. Let us introduce the following definition:

Definition 4 *An economy $\mathcal{E} = \{X, u_i, w_i, I\}$ will be called a developed economy, if there exists $(\lambda^*, x(\lambda^*)) \in \mathcal{C}_N$ such that $e(\lambda^*, w) = 0$ and $\mathcal{N}_N = W_{\lambda^*}, (x(\lambda^*))$.*

So, only developed economies can reach in equilibrium a social maximum welfare.

Note that in General Equilibrium theory there is not a definition of developed economy. This definition, given in the framework of the General Equilibrium theory, follows as a corollary of the Negishi approach, and relate efficiency, equilibrium and social welfare.

5 Recovering the preferences

As it is well known there exists a relation between the social weight of an individual and his marginal utility of imcome.“*The weight of a consumer being in inverse relation to the equilibrium marginal utility of income”* [Negishi, T. (1960)]. In this section we attempt to find a relation between the social weights of individuals and his marginal utility of consumption.

Suppose that the exchange economy is regular. This means that the jacobian of the excess utility function $J_\lambda e(\lambda, w)$ has rank equal to $n-1$ in each $\lambda : e(\lambda, w) = 0$. As it is well know from Walras law and from the homogeneous of degree zero property of the excess utility function [Accinelli, E. (1996)] it is possible to consider the reduced excess utility function $\bar{e}(\cdot, w) : R^{n-1} \to R^{n-1}$, and if the economy is regular, then the jacobian of this function has rang total, i.e.: $Rank[J_\lambda \bar{e}(\lambda, w)] = n-1$.

So, from the implicit function theorem, there exists $\lambda(w) \in C^1$ such that $\bar{e}(\lambda(w), w) = 0$. Taking derivatives with respect to w we obtain that

$$J_\lambda \bar{e}(\lambda(w), w)\lambda_w(w) + \bar{e}_w(\lambda(w), w) = 0 \tag{16}$$

where

$$\lambda_w = \begin{bmatrix} \frac{\partial \lambda_1}{\partial w_{11}} & \dots & \frac{\partial \lambda_1}{\partial w_{1l}} & \dots & \frac{\partial \lambda_1}{\partial w_{n1}} & \dots & \frac{\partial \lambda_1}{\partial w_{nl}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{\partial \lambda_{n-1}}{\partial w_{1l}} & \dots & \frac{\partial \lambda_{n-1}}{\partial w_{1l}} & \dots & \frac{\partial \lambda_{n-1}}{\partial w_{n1}} & \dots & \frac{\partial \lambda_{n-1}}{\partial w_{nl}}, \end{bmatrix}; \ \bar{e}_w = \begin{bmatrix} \frac{\partial \bar{e}_1}{\partial w_{11}} & \dots & \frac{\partial \bar{e}_1}{\partial w_{1l}} & \dots & \frac{\partial \bar{e}_1}{\partial w_{nl}} \\ \frac{\partial \bar{e}_2}{\partial w_{11}} & \dots & \frac{\partial \bar{e}_2}{\partial w_{1l}} & \dots & \frac{\partial \bar{e}_2}{\partial w_{nl}} \\ \dots & \dots & \dots & & \\ \frac{\partial \bar{e}_{(n-1)}}{\partial w_{11}} & \dots & \frac{\partial \bar{e}_1}{\partial w_{1l}} & \dots & \frac{\partial \bar{e}_{(n-1)}}{\partial w_{nl}} \end{bmatrix}$$

Note that

$$\frac{\partial \bar{e}_i}{\partial w_{jh}} = \begin{cases} \frac{\partial u_i}{\partial x_h} & if\ j = i \\ \\ 0 & \text{in other case} \end{cases} \quad \forall i, j \in \{1, ..., n\},\ h \in \{1, ..., n\}.$$

It follows that

$$\lambda_w(w) = (J_\lambda \bar{e}(\lambda(w), w))^{-1} \frac{d}{dx} U(x(\lambda(w))), \tag{17}$$

where

$$\bar{e}_w = \frac{d}{dx}U = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \dots & \frac{\partial u_1}{\partial x_l} & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & \frac{\partial u_2}{\partial x_1} & \dots & \frac{\partial u_2}{\partial x_l} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & 0 & \frac{\partial u_{n-1}}{\partial x_1} & \dots & \frac{\partial u_{n-1}}{\partial x_l} \end{bmatrix}$$

i.e the characteristics of the changes in the social equilibrium by reallocation of the endowments are strongly related with he marginal propensity to consume. Or equivalently income effects and preferences are strongly related in the social equilibrium manifold. In some sense this means that the characteristics of the manifold equilibrium are related with the preferences of the agents.

Suppose that the endowments change from w^0 to w^f then the change in the social equilibria is given (approximately) by:

$$\lambda(w^f) - \lambda(w^0) \simeq a\frac{d}{dx}U(x(\lambda^{w^0}))(w^f - w^0),$$

where $a = \left(J_\lambda \bar{e}(\lambda(w^0), w^0)\right)^{-1}$ i.e. the changes in the social weights are greater in consumer which marginal utility is greater than in the others.

One more question: Means this that, knowing the equilibrium manifold it is possible to recover the preferences of the economy?

The following example help to understand these considerations.

Example 1 *Consider a three-agents, three-goods economy, then the equation (17) take the form:*

$$\begin{bmatrix} \lambda_{1w_{11}} & \lambda_{1w_{12}} & \lambda_{1w_{13}} & \lambda_{1w_{21}} & \lambda_{1w_{22}} & \lambda_{1w_{23}} \\ \lambda_{2w_{12}} & \lambda_{2w_{12}} & \lambda_{1w_{23}} & \lambda_{2w_{21}} & \lambda_{2w_{22}} & \lambda_{2w_{23}} \end{bmatrix} =$$

$$= \begin{bmatrix} e_{1\lambda_1} & e_{1\lambda_2} \\ e_{2\lambda_1} & e_{2\lambda_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial u_1}{\partial x_{11}} & \frac{\partial u_1}{\partial x_{12}} & \frac{\partial u_1}{\partial x_{13}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial u_2}{\partial x_{12}} & \frac{\partial u_2}{\partial x_{22}} & \frac{\partial u_2}{\partial x_{23}} \end{bmatrix},$$

where $\lambda_{iw_{ij}}$ is de derivative of λ_i with respect to de endowment j of the $i-th$ consumer, $\frac{\partial u_i}{\partial x_{ij}}$ is the derivative of the utility of the $i-th$ consumer, respect to the $j-th$ variable, and $e_{i\lambda_j}$ is de derivative of the excess utility of the consumer i with respect to λ_j, $j = 1, 2$.

6 Conclusions

In this paper we introduce a relation between efficiency and social welfare. This relation is setting from the Negishi approach that consists in obtaining the Pareto optimal allocation by maximizing a social welfare function. The possibility that a central planner can choose the social weights cannot be implemented in a real economy. Nevertheless our approach gives place to new theoretical challenges, and definitions in the framework of the General Equilibrium theory. We also introduce new relations between social weights and social welfare. We give a partial answer to the question about in which cases the walrasian equilibrium allocations maximize social welfare. New questions about the connections between the rules to maximize a social utility function and order relations in the social weights set can be formulated. The question about the possibility of recovering the preferences considering the characteristic of the equilibrium social weights is the object of future researches.

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