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**Efficiency, egalitarianism, stability and social welfare in  
economics**

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# Efficiency, egalitarianism, stability and social welfare in economics

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## Abstract

The Pareto optimal concept does not concern with fairness or equality, it is a concept related to efficiency. In this paper, using techniques from the general equilibrium theory, we relate efficiency, fairness and stability of an economy.

Keywords: Fairness, efficiency, economics welfare

## Resumen

El concepto de óptimo de Pareto no se refiere a la equidad o la igualdad, es un concepto relacionado con la eficiencia. En este trabajo, utilizando técnicas propias de la teoría del equilibrio general, relacionamos la eficiencia, la equidad y la estabilidad de una economía.

Palabras clave: Equidad, eficiencia y bienestar económico

JEL: D4, D6

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## 1 Introduction

In this paper we discuss the relationship between Pareto optimality, social welfare and equality. We show that there exists an egalitarian and efficient allocation, ensuring at the same time, social stability. We argue that it is possible to obtain an stable, fair and efficient economy. We joint two classical and apparently different points of view. The point of view of the general equilibrium theory, following by Arrow [Arrow, K.], and on the other hand the point of view of the distributive justice, following by Sen [Sen, A.K.] and Rawls [Rawls, J. (1)]. These two and apparently antagonist points of view, can be summarized following the Negishi approach [Negishi, T.].

By efficiency we understand the efficiency in the Pareto optimality sense. The concept of equality considered in this work is close to the concept of the idea that John Rawls (1999) has called "equality of fair opportunity." Finally, stability is introduced as a concept of social stability of the economy, in the sense that the action of individuals who prefer to play in a non-cooperative way, can be blocked by the action of the rest of the society.

This work is organized as follows. In the next section we introduce the main characteristics of the economies considered. In section (3) we analyze the relationship between efficiency and social welfare. Next, in section (4) we introduce some considerations on the egalitarianism. In section (6) we introduce the definition of unequal economy and some considerations on the possibilities to reach egalitarian allocations in a decentralized way. Finally we give some conclusions.

## 2 The model

We consider an exchange economy composed by  $n$  consumers and  $l$  commodities

$$E = \{X_i, u_i, w_i, i \in I\}$$

where  $I = \{1, 2, \dots, n\}$  is an index set symbolizing the agents of the economy. We assume that the consumption set  $X_i$  is the same for all the agents and it is  $\mathbf{R}_+^l$ . The utility functions are strictly concave, monotone, and continuous functions. The endowments are denoted by  $w_i \in \mathbf{R}_+^l$ .

**Definition 1** An allocation  $x = (x_1, \dots, x_n)$  is a specification of a consumption bundle,  $x_i \in \mathbb{R}^l$  for each consumer  $i \in I$ .

Let us define the feasible set  $F \subset (\mathbb{R}^l)^n$ , as the set of consumption bundles,

$$F = \left\{ x = (x_1, \dots, x_n) : x_i \in \mathbb{R}_+^l \quad \forall i \in I, : \sum_{i=1}^n x_i \leq \sum_{i=1}^n w_i \right\},$$

and the utility possibility set:

$$U = \left\{ u \in \mathbb{R}^n : \text{there is an allocation } x \text{ such that } u_i \leq u_i(x_i), \forall i \in I \right\}$$

**Remark 1** (Notation) Given an allocation  $x = (x_1, \dots, x_n)$ , by  $u(x)$  we symbolize the vector  $(u_1(x_1), \dots, u_n(x_n))$ .

Note that under the assumptions of this work, the utility possibility set is convex. This result follows straightforward from the concavity of the utility functions because: If  $u^1, u^2 \in U$  then there exist  $x^1, x^2 \in F$  such that  $u_i^1 \leq u_i(x_i^1)$  and  $u_i^2 \leq u_i(x_i^2)$ . So,  $\alpha u_i^1 + (1-\alpha)u_i^2 \leq u_i(\alpha x_i^1 + (1-\alpha)x_i^2)$ ,  $\forall i \in \{1, 2, \dots, n\}$ . Since  $F$  is a convex set the affirmation holds.

**Definition 2** A feasible allocation  $x$  is Pareto optimal if there is no other allocation  $x'$  such that  $u_i(x'_i) \geq u_i(x_i)$  for all  $i \in I$  and  $u_k(x'_k) > u_k(x_k)$ .

From the previous definition it follows directly that the Pareto optimal concept does not concern with fairness. It is a concept related to efficiency in the sense that an allocation is Pareto optimal if there is no waste, i.e: it is not possible to improve any consumer's utility without making someone worse off.

By the definition of Pareto optimality, it follows that the Pareto optimal allocations must belong to the boundary of the utility possibility set. The boundary of this set will be denoted by  $UP$  and is defined by:

$$UP = \left\{ u \in U : \nexists u' \in U : u'_i \geq u_i \quad \forall i \in I, \text{ and } u'_k > u_k \text{ for some } k \in I \right\}$$

The next proposition is straightforward

**Proposition 1** *A feasible allocation  $x$  is a Pareto optimum if and only if  $(u_1(x_1), \dots, u_n(x_n)) \in UP$ .*

*Proof:* Since utilities are monotone and strictly concave, they are strictly monotone and then a feasible allocation  $x$  can be Pareto optimal if and only if the utility vector  $u = (u_1(x_1), \dots, u_n(x_n)) \in UP$ . •

### 3 Pareto optimality and social welfare optimum

In this section we discuss the relationship between the Pareto optimality concept and the maximization of a social welfare function.

We will consider a social welfare function particularly simple given by:  $U_\lambda : \mathbf{F} \rightarrow \mathbf{R}$ , and defined as:

$$U_\lambda(x) = \sum_{i=1}^n \lambda_i u_i(x_i) \quad (1)$$

where  $\lambda = (\lambda_1, \dots, \lambda_n)$  is fixed and can be considered as a vector of social weights. Since the social welfare function should be nondecreasing in the individual utility, we can consider  $\lambda \geq 0$ . Moreover we can assume that  $\lambda$  belongs to the  $n-1$  dimensional simplex  $\Delta^{n-1}$ . This function summarizes the social welfare associated to the allocation  $x$ , but certainly this social value changes if  $\lambda$  changes.

Note that if the utility vector  $u = (u_1, \dots, u_n)$  is associated with a Pareto optimal allocation  $x \in \mathbf{F}$ , being  $u_i = u_i(x_i)$  for each  $i \in \{1, \dots, n\}$  then,  $u$  is in the boundary of the possibility utility set. This observation suggests the next proposition:

**Proposition 2** *The set of Pareto optimal allocations is homeomorphic to the simplex  $\Delta^{n-1}$ .*

This is a consequence of the following lemma.

**Lemma 1** *If utilities  $u_i, i = 1, \dots, n$  are strictly concave, then  $UP$  is*

homeomorphic to the  $n-1$  simplex.

*Proof:* Consider the function  $\xi:UP \rightarrow \Delta$  defined by  $\xi(u) = \frac{1}{u_1 + \dots + u_n}u$ . Since  $\xi$

is a homeomorphism the result follows. •

This homeomorphism is shown in figure (1) (A) for two consumers, and (B) for the case of three consumers.

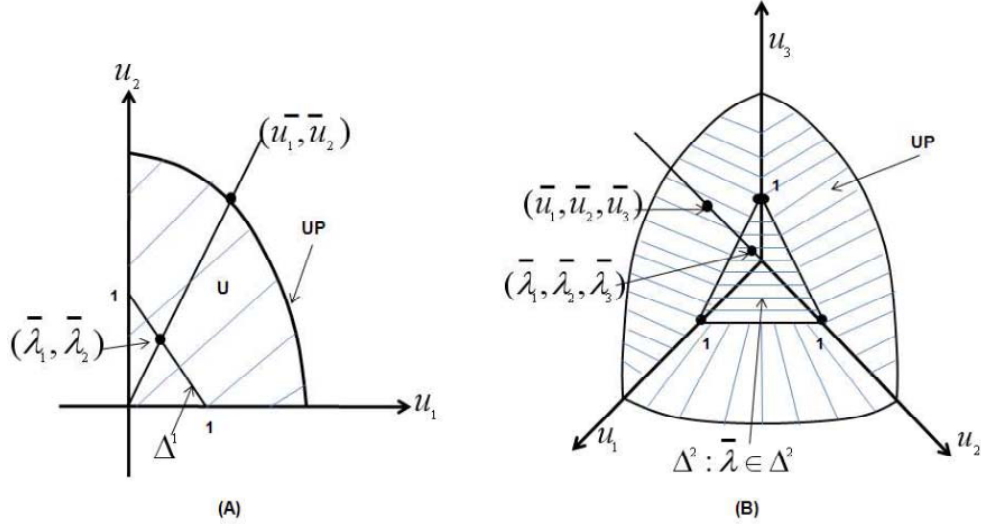


Figure 1: The homeomorphism between  $\Delta$  and  $UP$  for  $n=2$  and  $n=3$ .

The proposition (2) is a straightforward conclusion of this lemma.

*Proof of the proposition:* Let us symbolize by  $PO$  the set of Pareto optimal allocations, so for each  $u \in UP$  there exists  $x \in PO$  such that  $u = u(x)$  and reciprocally. Consider  $\phi:PO \rightarrow UP$  given by  $\phi(x) = u$  and  $\psi:PO \rightarrow \Delta^{n-1}$  given by  $\psi(x) = \xi(\phi(x)) = \lambda$ . •

If our interest is to find an allocation maximizing the social welfare, it is clear that this allocation must be chosen from the Pareto optimal allocations. Suppose that for a fixed  $\lambda \in \Delta^{n-1}$ , we consider the social utility function  $U_\lambda(x)$ , so it makes sense to select an allocations in  $F$  maximizing this function, i.e, solving the following maximization problem:

$$\max_{x \in F} U_\lambda(x) = \sum_{i=1}^n \lambda_i u_i(x_i) \quad (2)$$

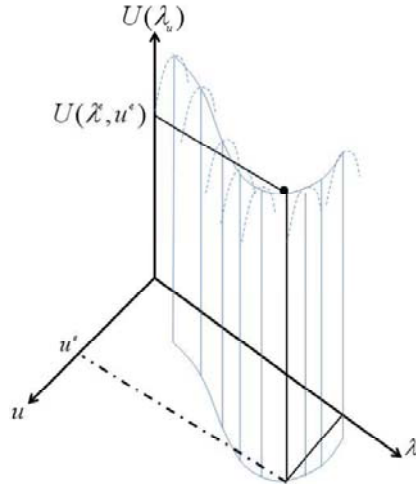


Figure 2: The Pareto optimal allocations and the equalitarian allocation

**Theorem 1** For each  $\lambda \in \Delta^{n-1}$  there exist a feasible allocation  $x^*$  solving the maximization problem 2 and this allocation is Pareto optimal. If utilities are strictly concave function this solution is unique.

*Proof:* For each  $\lambda \in \Delta^{n-1}$   $U_\lambda : F \rightarrow R$  is a continuous function, since  $F$  is closed and bounded, the function attain its maximum value is in this set. Now suppose that  $x^*$  is not Pareto optimal, then, there exist a feasible allocation  $\tilde{x}$  such that  $u(\tilde{x}) \geq u(x^*)$ ,  $u(\tilde{x}) \neq u(x^*)$  then  $\sum_{i=1}^n \bar{\lambda}_i u(\tilde{x}) > \sum_{i=1}^n \bar{\lambda}_i u(x^*)$ . • Finally, since a convex combination of strictly concave function is strictly concave the uniqueness of the maximum follows. •

The reciprocal of this theorem holds:

**Theorem 2** Given a Pareto optimal allocation  $\bar{x}$ , there exists a vector  $\bar{\lambda} \in \Delta^{n-1}$  such that  $\bar{x}$  solves the maximization problem:

$$\max_{x \in F} \sum_{i=1}^n \bar{\lambda}_i u_i(x)$$

ie:  $U_{\bar{\lambda}}(\bar{x}) \geq U_{\bar{\lambda}}(x) \forall x \in F$ . Moreover if utilities are strictly concave functions then  $\bar{\lambda}$  is the unique element in the simplex verifying  $\bar{\lambda} u(\bar{x}) \geq \bar{\lambda} u(x) \forall x \in F$ .

*Proof:* If the allocation  $\bar{x}$  is Pareto optimal then  $\bar{u} = u(\bar{x})$  is in the boundary of the

utility possibility set. Since this is a convex set, by the supporting hyperplane theorem, there exists  $\bar{\lambda} \neq 0$  such that that  $\bar{\lambda} \bar{u} \geq \bar{\lambda} u \forall u \in UP$ . On the other hand  $\lambda \in \mathbb{R}_+^n$  because if any  $\lambda_i < 0$  then considering the  $u \notin UP$  with  $u_i$  big enough and  $0 \leq u_j \leq \varepsilon, \forall j \neq i$  and  $0 < \varepsilon$  and small enough then  $\lambda(\bar{u} - u) > 0$ , but this is not possible for  $u \notin UP$ . •

#### 4 Efficiency and egalitarian

As we have shown in the previous section, given a vector  $\lambda \in \Delta^{n-1}$  there exists a Pareto optimal allocation  $x^*(\lambda)$  such that:

$$U_\lambda(x^*(\lambda)) \geq U_\lambda(x) \forall x \in \mathbf{F} \quad (3)$$

Let us introduce the function  $\tilde{U} : \Delta^{n-1} \rightarrow \mathbb{R}$  defined by:

$$\tilde{U}(\lambda) = U(\lambda, x^*(\lambda)) = \sum_{i=1}^n \lambda_i u_i(x_i^*(\lambda))$$

where  $x(\lambda)$  is the Pareto optimal allocation such that  $u \in UP$  verify  $u = u(x^*(\lambda))$  being  $\lambda = \xi(u)$ .

Now we introduce some consideration on the egalitarian allocation,  $x^e$ , understanding as egalitarian, a Pareto optimal allocation such that every individual attains the same level of utility.

**Proposition 3** *The egalitarian allocation  $x^e$  solving  $u_i(x_i^e) = u^e, \forall i \in I$  is the Pareto optimal allocation corresponding to the solution of the minimization problem:*

$$\min_{\lambda \in \Delta^{n-1}} U(\lambda, x^*(\lambda)) = \sum_{i=1}^n \lambda_i u_i(x_i^*(\lambda))$$

*Proof:* In [Accinelli, E.; Brida, G. Plata, L.; Puchet. M] is shown that the function  $\tilde{U}(\lambda) = U(\lambda, x(\lambda))$  is strictly convex. So the first order condition is a necessary and sufficient condition for minimization. Let  $\lambda^e$  be the solution of this problem. It follows that  $u_i(x_i(\lambda^e)) = u_j(x_j(\lambda^e)) = U(\lambda^e, x^*(\lambda^e)) = \tilde{U}(\lambda^e) \forall i, j \in I$ . •



**Proposition 4** Let  $x^e$  the egalitarian solution. There exist  $\lambda^e \in \Delta^{n-1}$  such that  $\lambda^e u(x^e) \geq \lambda^e u(x) \forall x \in F$ , and  $\lambda^e = (\frac{1}{n}, \dots, \frac{1}{n})$

*Proof:* Since  $x^e$  is feasible and Pareto optimal allocation, from theorem (2) such  $\lambda^e$  exists. To see the second part, suppose that  $\lambda_j^e \geq \lambda_i^e \forall i = 1, \dots, n$  strictly greater for at least one coordinate. Then the following vector of utilities is in the boundary of the utility possibility set:

$$u_\varepsilon = (u_1^e - \frac{\varepsilon}{n-1}, \dots, u_i^e + a\varepsilon, \dots, u_n^e - \frac{\varepsilon}{n-1})$$

$a$  and  $\varepsilon$  are chosen so that  $\lambda^e = \xi^{-1}(u^e)$ . So,  $\lambda^e u_\varepsilon > \lambda^e u^e$  this is absurd for definition on  $\lambda^e$ . •

## 5 Stability of the egalitarian solution

Note that among all the efficient allocations the egalitarian solution (corresponding to the minimum value of  $U(\lambda, x^*(\lambda))$ ) is the only stable solution. Because any change in the parameters of the economy, imply that one agent in the economy attains a high level of utility but in detriment of the rest of the society. See figure (3). So, after any perturbation in the fundamentals of the egalitarian economy, the rest of the society will push to return to the egalitarian situation. In this sense it is possible to say that the egalitarian solution corresponds to an efficient and consensual wealth distribution.

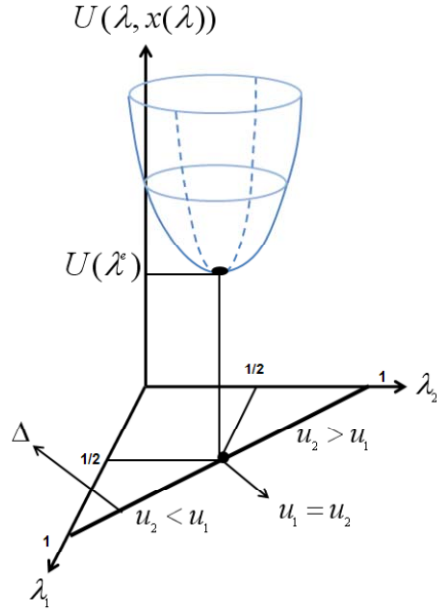


Figure 3: The equalitarian allocation

The egalitarian distribution can be attained in a decentralized way, if and only if the distribution of the initial endowments allow that this allocation can be attained as a Walrasian allocation.

The economy supporting such allocation as a Walrasian equilibrium is stable, because as we said, any perturbation of the fundamentals of the economy, makes that some individuals attain high level of welfare in detriment of the rest of the society. Note that the egalitarian allocation implies equal level of happiness, not necessarily the same bundle set for every consumer. If the egalitarian allocation is reached as a Walrasian allocation, then the social weights of all consumers are the same, the intuition behind this affirmation is that the different social groups have similar economic power. So, following [Barbosa, P.; Jovanovic, B.; Spiegel, M.] this situation imply that: " An economy remains in force so long as no party wishes to defect to the noncooperative situation, and it is reinstated as soon as each party finds it to its advantage to revert to cooperation" .

From theorem (1) for each  $\bar{\lambda} \in \Delta^{n-1}$  there exist a Pareto optimal allocation  $\bar{x}$  such that  $\bar{\lambda}u(\bar{x}) \geq \bar{\lambda}u(x) \forall x \in F$ . So, there exist a function  $\phi: \Delta^{n-1} \rightarrow UP$  defined by  $\phi(\bar{\lambda}) = u(\bar{x})$  making possible to define the following path of the efficiency:

**Definition 3** The path  $NPU = \{(\lambda, \phi(\lambda)), \lambda \in \Delta^{n-1}\}$  will be called the Negishi utility path.

This definition is equivalent to the definition of the Negishi path given in [Accinelli, E. Hernandez, R.; Plata, L.].

Along the Negishi path we find the set of pairs of  $(\lambda, \phi(\lambda)) \in \Delta^{n-1} \times \text{UP}$  corresponding to each Pareto optimal allocation.

Consider the function  $\tilde{U} : \text{NPU} \rightarrow R$  defined by

$$\tilde{U}(\lambda, \phi(\lambda)) = \sum_{i=1}^n \lambda_i u_i^\lambda$$

where  $\phi(\lambda) = u^\lambda = (u_1^\lambda, \dots, u_n^\lambda)$ . This function, defined along the Negishi utility path reach it minimum at  $\lambda^e$  i.e:

$$\tilde{U}(\lambda, \phi(\lambda)) = \tilde{U}(\lambda, u^\lambda) \geq \tilde{U}(\lambda^e, u^e) = \tilde{U}(\lambda^e, \phi(\lambda^e)), \forall \lambda \in \Delta^{n-1}.$$

John Rawls's theory of justice, it is asserted that institutions and practices should be arranged so that the worst off are as well off over the long run as possible, they work to the maximal advantage of the worst off members of society, (see [Rawls, J. (1)] and [Rawls, J. (2)]). Precisely, the utility obtained from the egalitarian allocation corresponds to the solution of maximizing the utility of those individuals who achieve worse results, i.e.,

$$u^e = \max_{u \in \text{UP}} \{ \min \{ u_1, \dots, u_n \} \}$$

Following [Bowles, S. and Herbert, G.] that more equal countries have more rapid rates of economic growth could well be accounted for by a statistical association between measures of equality and unmeasured causes of economic growth. This observation does not imply, that equality per se promotes high levels of economic performance, but egalitarian policies are compatible with the rapid growth of productivity. The capitalist countries taken as a whole have grown faster under the aegis of the post Second World War than in any other period, and in this was the period of ascendent welfare state and social democracy.

According with the above statements, those countries with higher growth rates, correspond to which social weight distribution is more unequal. Conversely countries with greater social justice, would be those in which the social weight distribution closer to the egalitarian.

## 6 Welfare and markets

The main question of this section is if a society based on free markets can attain the egalitarian allocation.

The agents go to the market with the purpose of finding a bundle set preferable to their endowments, i.e., the  $i$ -th agent go to the market to find a bundle set  $x_i \in \mathbf{R}_+^l : u_i(x_i) \geq u_i(w_i), i = 1, 2, \dots, n$ . Only an allocation being part of a Walrasian equilibrium can be attained in a decentralized way. From the first theorem of the welfare such allocations  $x \in \mathbf{R}^m$  are Pareto optimal, and given the rationality of the agents, these allocation must verify that  $u_i(x_i) \geq u_i(w_i), i = 1, \dots, n$ . We denote by RPO the set of allocations  $x \in OP$  such that  $u_i(x_i) \geq u_i(w_i), \forall i = 1, \dots, n$ . The corresponding levels of utility for this allocation are given by:

$$\text{RUP} = \{u \in \text{UP} : u_i \geq u_i(x_i) \forall i = 1, 2, \dots, n\}$$

see figure (4).

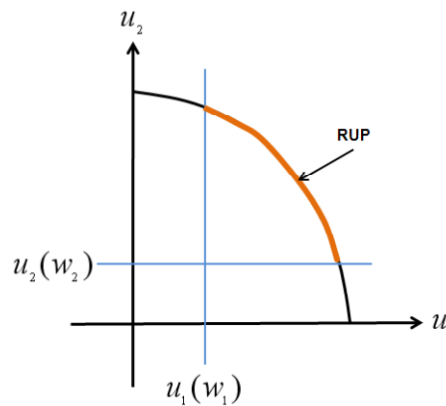


Figure 4: Rational Pareto optimal allocations

We said that given an economy  $E$  a feasible allocation  $x^w$  is Walrasian if there exists a set of prices  $p \in \mathbf{R}_+^l$  such that the pair  $(x^w, p)$  is a Walrasian equilibrium for the economy  $E$ . We symbolize by  $W_E$  the set of Walrasian allocations of a given economy  $E$ .

The first theorem of welfare economics establishes a relationship between Walrasian allocations and Pareto optimal allocations. Since the only of these Pareto optimal allocations can be achieved in a decentralized way, i.e., by the unique action of the laws of economics,

are the Walrasian allocations, the possible levels of utilities, attainable in a given economy depend on the distribution of initial endowments. So, it is possible that for a given economy, with a very unequal distribution of the initial endowments can not be attained by the only action of the market law the egalitarian allocation.

The second welfare theorem says that for any Pareto optimal allocation  $x^P$  there exists a vector of prices  $p$  such that the pair  $(p, x^P)$  is an equilibrium with transfer payments  $t_i = p(x_i^o - w^i)$ . In other words, a benevolent social planner after to transfer wealth, can make that the economy, acting under its own laws, attain a socially desirable Pareto optimal allocation in a decentralized way, i.e., as a Walrasian equilibrium.

Let us define an unequal economy:

**Definition 4** *An economy  $E$  is unequal if the egalitarian allocation  $x^e$  is not a rational Pareto optimal allocation. That is,  $x^e \notin RPO$ . Corresponds to an economy where the distribution of wealth is very unevenly.*

So, a unequal economy, whose agents are rational, can not attain an egalitarian distribution of wealth by the only action of the markets, see figure 5). To attain certain degree of social justice, starting with an excessively unequal distribution of endowments, imply the participation of a central planer able to implement a set of economic policy measures to this end. This affirmation can be summarized in the next proposition:

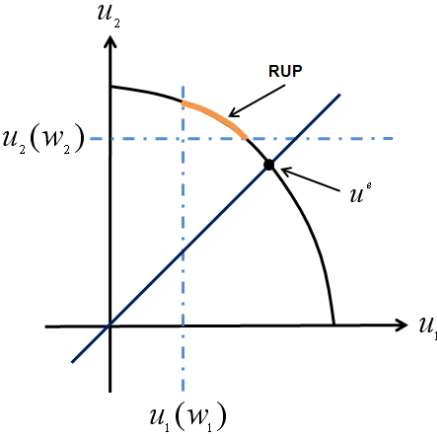


Figure 5: An unequal economy

**Proposition 5** *Given an unequal economy, the egalitarian distribution  $x^e$  can not be attained in a decentralized way.*

*Proof:* Since  $x^e \notin \text{RPO}$  there is a neighborhood  $V_{x^e} \subset \mathbb{R}^{ln}$  of this allocation such that no allocation in  $UP \cap V_{x^e}$  can be a Walrasian allocation. •

**Corolary 1** *In an unequal economy, there exists  $\varepsilon > 0$  such that the levels of utility  $u^w$  corresponding with a Walrasian allocation verify the inequality:  $|u^w - u^e| > \varepsilon$ .*

**Corolary 2** *In an unequal economy,  $u^e \notin \text{RUP}$*

Let  $x^w \in W_E$ , be a walrasian allocation, the ratio  $\frac{u_i(x_i^w)}{u_i(w_i)}$  measures the relative value that the  $i$ -th consumer assigns to the market allocation, and the ratio  $\frac{u_i(x_i^e)}{u_i(w_i)}$  measures the relative value that the  $i$ -th consumer assigns to the egalitarian allocation. A consumer prefer the Walrasian allocation  $x^w$  to the egalitarian allocation  $x^e$  if and only if  $\frac{u_i^w}{u_i(w_i)} > \frac{u_i^e}{u_i(w_i)}$  where  $u_i^w = u_i(x_i^w)$  and  $u_i^e = u_i(x_i^e)$ .

Let us define the subset  $U^w \subset U$  where

$$U^w = \left\{ u^w \in \mathbb{R}^n : \text{there exists } x^w \in W_E \text{ such that } u^w = u(x^w) \right\}$$

This subset captures the attainable vectors of utilities levels that can be obtained by means of a Walrasian allocation.

**Definition 5** *The following index measures how far a given economy  $E_{im}$  is to achieve in a decentralized way an equal distribution:*

$$I_E = \min_{u^w \in U^w} \sum_{i=1}^n \frac{|u_i^{w_i} - u_i^e|}{u_i^{w_i}}$$

If for a given economy, this index is positive, then the equal distribution can be achieved only after transfers.

Since utilities are not observable we can measure the degree of inequality of an economy from de following index:

**Definition 6** *The following index measures how far a given economy  $E_{in}$  is to achieve in a decentralized way an equal distribution:*

$$I_E = \min_{x^w \in W_E} \sum_{i=1}^n \frac{|x_i^w - x_i^e|}{|x_i^{w_i}|}$$

The following proposition characterizes an unequal economy:

**Proposition 6** *Let  $E$  be an economy which endowments are  $W = (w_1, \dots, w_n)$ . The economy is unequal if and only if there exists an individual such that  $u_i(w_i) > u_i^e = \tilde{U}(\lambda^e)$ .*

*Proof:* Since the Walrasian allocation  $x^w$  corresponding with this economy, must verify that  $u_i(x_i^w) \geq u_i(w_i) > u_i^e$ , then the egalitarian allocation can not be a Walrasian allocation for  $E$ . •

This proposition is shown in figure (5). Note that the definition of unequal economy does not depend on the utilities representing the preferences of the consumers.

In accordance with propositions (5) and (6) economies with a high number of individual under the poverty line can not attain high levels of welfare, by the only action of the markets. However the second welfare theorem says that under transference it is possible to obtain a vector of prices supporting this allocation as a Walrasian allocation. So, to obtain an egalitarian economy starting from a unequal economy it is necessary to implement a set of measures of political economy to attain this objective. According to the second welfare theorem, given a Pareto optimal allocation, there exist a set of process supporting this allocation. Recall that a set of prices  $p \in R_+^I$  support the allocation  $x$  if for each allocation  $y$  such that  $u_i(y_i) > u_i(x_i)$  then  $py_i > px_i \forall i \in I$ . Then the pair  $(x, p)$  is a walrasian equilibrium under transferences.

Note that at the same time that an economy approach the egalitarian solution the social weights of the different agents tend to be equal.

## 7 Conclusion

Free markets ensure efficiency but in some cases they can not ensure egalitarian allocation. In some cases the only Walrasian allocations possible to be reached by the only action of the free markets have associate very unequal levels of happiness. Obviously, this situation give place to a very unstable society, where more unhappy people can recruit for potential violent movements.

In this cases the participation of a central planner can introduce stability in the economy, if he is able to implement measures diminishing inequality. However, as is increasingly recognized, the intervention of a central authority to alter the distribution of the income can be accompanied of heavy political and economic costs. On the other hand, those who would harmed by these policies (the wealthy) can organize effective political opposition.

An alternative policy to that directly alter the distribution of wealth may be to encourage investment in technology and human capital increasing in this way the endowments of the workers. Technologically developed firms get more productivity and also pay higher wages to their workers, in particular for skilled workers.

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