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On the dynamics and effects of corruption on environmental protection

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Abstract

This paper studies the joint dynamics of corruption and pollution in a model of evolutionary game theory, where firms face a given pollution standard and the government must check the compliance to this standard by means of public officials who can be honest or not. A novelty of our paper is that officials decide to be honest or not by imitation, while firms are assumed to be inter-temporal profit maximizers. One of the main findings of the paper is that one possible " bad" outcome characterised by a whole society of polluting firms and corrupt officers can be sustained by rational agents who learn by imitation, despite the existence of multiplicity of equilibria of a perfectly honest population and a more realistic simultaneous presence of honest and dishonest agents. Furthermore, we show that the firm's discount rate is an important decision factor that influences the environmental pollution.

Keywords: Bribes and corruption; Environmental quality restrictions; Games and Economics; Imitation and evolutionary dynamics.
JEL codes: C70, C72, D21, K42, L21.

Resumen

En este trabajo se estudia la dinámica conjunta de la corrupción y la contaminación en el marco de la teoría de juegos evolutiva, donde las empresas se enfrentan a regulaciones sobre la contaminación y el gobierno debe verificar el cumplimiento de las normas impuestas, por medio de los funcionarios públicos que pueden ser honestos o no. Una novedad de nuestro trabajo es que los funcionarios deciden ser honestos o no por imitación, mientras que las empresas se supone que son intertemporalmente maximizadores de beneficio. Una de las principales conclusiones del estudio es que es posible un "mal" resultado desde el punto de vista de la sociedad, en el que se obtienen empresas contaminantes y funcionarios corruptos como resultado de la acción racional de agentes que actúan imitando el comportamiento de otros. No obstante, se concluye en la existencia de multiplicidad de equilibrios, uno de los cuales corresponde a una presencia simultánea, más realista, de agentes honestos y deshonestos. Además, se muestra que la tasa de descuento de la empresa es un factor de decisión importante que influye en la contaminación ambiental.

Palabras clave: sobornos y corrupción; restricciones medioambientales cualitativas; juegos y economía; imitación y dinámicas evolutivas.

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1 Introduction

Few studies have been devoted to model the dynamics of firms' bribing behavior and corruption driven by imitation, and in fact most studies are static instead. Little effort has been made to model the equilibrium level of bribery in an economy by taking into consideration both the macro environment and the micro-bribing behavior. Corrupt behavior is defined as bribes paid by firms to public officials (auditors).

Related references on corrupt behavior began with Tirole's seminal paper (1996) as one of the first attempts to model group reputation and the persistence of corruption as an aggregate of individual reputations. He studies the joint dynamics of individual and collective reputations and derives conditions to rebuild group reputations. In his work, group reputation is modelled as an aggregate of individual reputations, and new members joining a group " inherit" the good or bad reputation of the coalition. Stereotypes about the expected quality of a group are history dependent since collective reputation is a long term, path dependent and long-lasting process because new members inherit the reputation of the elders. Despite the model by Tirole, few studies have been devoted to model firms' bribing behavior (see Svensson, 2005, for a literature review).

Mishra (2006) considers a group of firms facing a certain pollution standard to illustrate how pervasive corruption can become a social norm. He shows that corruption or non-compliant behavior can be the equilibrium outcome in some cases and in such situations, corruption is the norm rather than deviant behavior.

Carillo (2000) develops a dynamic model of corruption in which agents are aware of their " propensity for corruption" and their clients choose an optimal level of bribe to offer. Such a framework provides an explanation for different implicit prices for illegal services (bribes or kick-backs) for similar countries (or organizations within similar countries), based on an analysis of clients' reaction.

Fredriksson and Svensson (2003) develop a theory of environmental policy formation, taking into consideration the degrees of corruptibility and political turbulence. They find an empirical interaction between corruption and political instability, i.e. political instability has a negative effect on the stringency of environmental regulations if the level of corruption is low, but a positive effect when the degree of corruption is high.

Wydick (2008) argues that in a free market, firms with well-defined property rights have no incentives to bribe public officials. However, if the government uses monopoly power to interfere with and restrict the market, then firms may be forced to bribe public officials. Firms face the "prisoner’s dilemma" in the sense that if all the firms refuse to bribe, they will all be better off, but since a single deviation will make the deviant firm better off when the other players are playing honestly, every firm realises that the others will cheat and must therefore bribe to remain competitive and they will be collectively worse off as a result [see, e.g., (Shleifer & Vishny, 1994); (Rose-Ackerman, 1997)]. Hardin (1968), in his seminal essay, calls it the "tragedy of the commons". Fortunately, in reality, we observe that the tragedy of the commons does not occur everywhere. In some societies, firms paying bribes to the government are very rare (e.g., the Scandinavian countries), although other negative examples of widespread corruption exist. Shleifer and Vishny (1993) point out two different types of corruption: (i) "Corruption without theft" where the corrupt official accepts a bribe to provide whatever service, but then turns over the legal price of the service to the government and (ii) "Corruption with theft" in which the corrupt officer accepts the bribe, but then doesn’t turn over anything to the government at all. According to Whydick (2008), this latter type of corruption is rampant and hard to stop, and is the type of corruption we consider in this paper.

Our aim is to study the joint dynamics of corruption and polluting choices undertaken by firms, with the novel assumption in this contex that the learning process is simply imitation, in a context of environmental protection regulation. Our approach comes from evolutionary game theory and dynamic optimisation[4]. The hypothesis of evolutionary dynamics driven by imitation helps us to understand the strategic foundations of the stable corrupt behaviour equilibrium. In the real world indeed, pervasive corruption sometimes is a “social norm”, although some other opposite examples of almost absence of corruption exist (according to Transparency International on the Global Corruption Barometer[5] New Zealand, Singapore or Finland are very little corrupted). In Mexico, for instance, corruption is widespread at all levels in public offices, and this behaviour is sustained by imitation, because people get corrupted because the others are. Corruption comes at all levels in Mexico as a kind of "cultural behavior", since the word for bribe, mordida, literally means bite, and getting bitten in Mexico is regrettably common [see Wydick (2008:1)]. In Mexico the mordida permeates every level of society and institutions where individuals act because it is a norm and they just do what the others are doing (to be

[4] Evolutionary analysis is well-documented in the game theory literature (see Weibull, 1995).
[5] Available at: http://www.transparency.org/policy_research/surveys_indices/gcb/2010

corrupt or not). Bribes in Mexico are common and indeed are often necessary for obtaining business licenses and other types of permits. A popular Mexican saying: "*el que no transa no avanza, who does not corrupt does not move on*", highlights how corruption is fundamental for personal attainament.

The point of departure of our evolutionary model is that peple's beliefs are not always rational. In general, in evolutionary games strategies emerge from a trial-and-error learning process according to which players find that some strategies perform better than others, and afterwards, they decide to adopt - or simply imitate - them[6].

We assume that firms face a given pollution standard, decided by the government. The government exercises a control over those firms through public officials, who have to check the "quality" of the firm by writing down a report stating whether the level of pollution produced by the firm itself is above or lower the standard. A negative report (a report stating a level of pollution above the standard) implies a fine to be paid by the firm to the government. This fact may induce a corrupt firm which does not respect the standard to offer a bribe to the officer, who accepts it if he is corrupt as well or refuse it if he is not. If a firm instead respect the standard, this does not mean that the fine is avoided, because if the officer is corrupt, he may thread to write down a negative report and ask a bribe. It is assumed that even though the firm can appeal to the court against the unfair report, this requires a long bureaucratic process so the firm strictly prefers to pay the requested bribe.

It will be clear later that one possible "bad" outcome characterised by a whole society of polluting firms and corrupt officers can be sustained by rational agents who learn by imitation, despite the existence of multiplicity of equilibria of a perfectly honest population and a more realistic simultaneous presence of honest and dishonest agents.

The remainder of this paper is organized as follows. Section 2 presents a one-shot 2x2 game to model firms' pollution decisions and officers' decisions about to be corrupt. Section 3 develops a model of evolutionary dynamics of officers' imitative behavior about to accept (or ask for) a bribe or not. We consider that official behavior is driven by imitation of the most successful. Section 4 develops a model for the dynamics of payoff-maximizing firms. Subsequently, we consider a dynamic decision problem for a firm facing intertermporal externalities from pollution. Section 5 concludes the paper.

[6] See Sanditov (2006) for a definition of imitative behaviour.

2 The one-shot game

This section introduces a 2×2 game between an inspection official and a firm. As previously mentioned, the game assumes two types of individuals, firms and public officials, who can be either corrupt or not. We assume that there exists a regulatory institution, namely a court or environmental authority, which decides a given pollution standard (assumed to be zero for convenience) that has to be respected by the firms. This authority checks the compliance to this standard by means of public officials who have the duty to inspect and measure the level of emissions produced by each firm, and write down a report declaring whether the firm respects the standard or not.

Each player decides to be corrupt or honest: a corrupt firm does not respect the pollution standard decided by the government and, when inspected by a public official, offers a bribe to avoid the fine. A corrupt officer, instead, accepts the bribe when offered by a corrupt firm, or asks for one when he inspects an honest firm (meaning a firm which respects the standard), by threading her to write down an unfair report and let her pay the fine. Even though the firm can appeal to the court against the unfair report, this requires a long bureaucratic process so we assume that the firm strictly prefers to pay the requested bribe.

An honest officer refuses the bribe offered by a corrupt firm, and writes down fair reports, irrespective of the fact he is meeting an honest firm or not.

Moreover, consider that:

• There are strategic complementarities, i.e. a polluting $p-$firm prefers matching a dishonest $d-$official and a non-polluting $n-$firm prefers matching an honest $h-$official.

• With some probability the $d-$official is caught and fined by court. If the official is detected by the court, he is charged a fine $M > 0$. The probability to detect a dishonest officer is denoted by $P \in [0,1]$.

• For each firm inspected, the officer receives a monetary reward $W > 0$, and bears an effort $e \le W$ which does not change according to the level of emissions produced or whether the firm is corrupt or not.

• Nonpolluting firms pay a fixed environmental cost of production, denoted by $C > 0$, which represents the additional cost of buying newer an environmentally friendly machinery, while polluting firms do not consider this cost.

• A polluting firm found guilty by the court is charged a fine denoted by $\mathsf{F} > 0$. A nonpolluting firm inspected by an honest official does not pay the fine.

• A polluting firm inspected by a dishonest official pays a bribe R and avoids the fine. Notice that the bribe R is a monetary quantity, $R = \theta \mathsf{F} > 0$, with $\theta \in (0,1)$.

• A non-polluting firm inspected by a dishonest official must pay a bribe, $r_d = \theta W > 0$ with $\theta \in (0,1)$, for having a fair report. That is non-polluting firms must pay a bribe when inspection is done by a dishonest official.

The 2×2 game between the inspection officer, O, and a firm, F, is introduced in the following definition:

Definition 1 *The normal-form representation of this game is the following payoff matrix:*

O/F	p	n
d	$W + R - e - PM,\ \pi_p - R$	$W + r_d - e - PM,\ \pi_n - r_d - C$
h	$W - e,\ \pi_p - F$	$W - e,\ \pi_n - C$

where $\pi_p > 0$ is the gross-payoff (revenues) of the polluting firm and $\pi_n > 0$ is the revenues of the non-polluting firm. The natural choice of parameters $R - PM \geq 0$ and $\pi_p - R \geq \pi_n - r_d - C$, makes it a coordination game.

The game can be represented as a two-population normal form game denoted by the list:

$$\Gamma = \langle (\{F, O\}); i \in \{p, n; h, d\}; E(i) \rangle.$$

for each population of officers and firms $\{O, F\}$ with their respective vector of strategies for each $i-$strategic player, and respective expected payoffs $E(i)$.

Let us denote by $O=(O_h, O_d)\in\Delta^O$ the profile distribution of officers' type in a given period of time t_0, where O_h is the share of honest officers, and O_d is the share of dishonest officers. At the same period of time the profile distribution of firms' type is given by $F=(f_p, f_n)\in\Delta^F$ where f_p is the share of polluting firms, and f_n is the share of non-polluting firms. Hence the strategy distribution of each population is given by:

$$\Delta^O=\{O\in R_+^2 : O_h+O_d=1\}.$$

$$\Delta^F=\{F\in R_+^2 : f_p+f_n=1\}.$$

Note that the expected payoff of a non-polluting firm is given by:

$$E(f_n\,|\,O)=[\pi_{np}-C]O_h+[\pi_n-C-r_d]O_d \tag{1}$$

and the expected payoff of a polluting firm is given by:

$$E(f_p\,|\,O)=[\pi_p-\mathsf{F}]O_h+[\pi_p-R]O. \tag{2}$$

where π_p is the profit of a polluting firm. Firms prefer polluting if $E(f_p\,|\,O)>E(f_n\,|\,O)$ and this happens if the share of honest officials is not large enough, i.e.

$$O_h<\overline{O_h}=\frac{\pi_p-\pi_n+C+r_d-R}{\mathsf{F}+r_d-R}.$$

Moreover, if $\pi_p-\mathsf{F}>\pi-C$ firms prefer to be polluting.

Similarly, the expected payoffs of the honest, $h-$official, and dishonest, $d-$official, is given by:

$$E(O_h\,|\,F)=W-e.$$

$$E(O_d\,|\,F)=W+R+r_d-e-PM+f_n(r_d-R).$$

Note that $E(O_h\,|\,F)\geq 0$ since $W\geq e$ and $E(O_d\,|\,F)\geq 0$ if $f_n\geq\dfrac{e+P(d)M-(W+R)}{r_d-R}$, since we are considering the case of non-negative expected payoffs. Therefore, officials prefer to be $h-$official if $E(O_h\,|\,F)>E(O_d\,|\,F)$, and it happens if the share of non-polluting firms is large enough, i.e.:

$$f_n > f_n = \frac{R + r_{dh} - P(d)M}{R - r_{dh}}.$$

And this happens if either the fine for a dishonest official or the probability that the court monitoring a dishonest behavior increase.

Remark 1 *For the above one-shot game (Definition 1), in order to eradicate polluting firms and dishonest officials, the punishments (fines* F *and* PM *) must be greater than the environmental costs of production and the bribes (*C*,* R *and* r_d *).*

Of course, under the quantitative relationships between payoffs described above, we can state that:

Remark 2 *The game* Γ *has two pure Nash equilibria. One is a high-compliance equilibrium with no firms choosing to pollute and officials remaining honest,* $(n, h) = (f_n, f_p; O_h, O_d) = (1,0; 1,0)$. *The other is the low-compliance equilibrium where all the officers are dishonest and all the firms choose to pollute,* $(p, d) = (f_n, f_p; O_h, O_d) = (0,1; 0,1)$. *There is also a mixed strategy Nash equilibrium given by:*

$$(\overline{O}_h, (1 - \overline{O}_h);\ f_n, (1 - f_n)). \tag{3}$$

where firms and officials are indifferent between to be corrupt or not.

Hence a firm's decision regarding whether to be polluting or not will depend on the probability to encounter an honest officer. Hence the "evolution" of honesty amongst the officers over time will affect the level of pollution.

3 The officers' imitative behavior

This section presents the key innovative feature of the paper. We study an analysis of corruption amongst officers by using imitative dynamics, since we argue that "imitation" of corrupt and successful strategy has a lot to do with the spread and persistence of corruption. From this perspective, we present the evolutionary dynamics of corruption driven by imitative behavior.

To explain why individuals imitate we should think of it as a kind of rational behavior (see Accinelli et al., 2010). Imitation results in agents performing a spectrum of tasks "as others do". We assume that occasionally each individual in a finite population gets an impulse to revise her (pure) strategy choice (be corrupted or non-corrupted). There are two basic elements in imitation theory. The first is a specification of the time rate at which individuals in the population review their current strategy choice. This rate may depend on the current performance of the agent's pure strategy and on other aspects of the current population state. The second element is a specification of the choice probabilities of a reviewing individual. The probability an i-strategist will switch to some pure strategy j may depend on the current performance of these strategies and other aspects of the current population's state. If these impulses arrive according to i.i.d. Poisson processe, then the probability of simultaneous impulses is zero, and the aggregate process is also a Poisson process. Moreover, the intensity of the aggregate process is just the sum of the intensities of the individual processes. If the population is large, then one may approximate the aggregate process by deterministic flows given by the expected payoffs from corruptive and non-corruptive behaviors.

Björnerstedt and Weibull (1996) study a number of such models, where individuals who revise may imitate other agents in their population of players, and show that a number of payoff-positive selection dynamics, including the replicator dynamics, may be so derived. In particular, if an individual's revision rate is linearly decreasing in the expected payoff of her strategy (or of the individual's latest payoff realization), then the intensity of each pure strategy's Poisson process will be proportional to its population share, and the proportionality factor will be linearly decreasing in its expected payoff. If every revising agent selects her future strategy by imitating a randomly drawn agent in their own player population,[7] then the resulting flow approximation is the replicator dynamics.

In the sequel, we consider that officials follow an imitative behavior of the best performed strategy given a fixed distribution of the share of non-polluting and polluting firms. A reviewer official i is willing to review her current strategy $i = \{h, d\}$, sometimes resulting in a change on it, with probability $r_i(O) \in [0,1]$. $r_i(O)$ is then the time rate at

[7] Evolutionary game theory considers populations of decision makers, while analysing the player profiles within these populations, instead of single players. We can therefore identify a population game, where N large populations strategically interact, as an $N-$player form game, where each player has a large population behind him (see Hofbauer and Sigmund, 2002).

which officials review their strategy choice. This probability depends on the actual distribution of the honest and dishonest officers and in the benefits associated with her current behavior.[8] It is natural to assume that the likelihood that an official will be willing to change her current strategy depends inversely with the performance of her current behavior. Having opted for a change, the official will adopt a strategy followed by the first successful person met from the population (her neighbour), i.e. there is a probability $p_{ij}(O) \in [0,1]$, that a reviewing $i-$official really switch to some pure strategy $j = \{h, d\}$, $j \neq i$. Assuming a continuum of officers, independence of switches across officials' same type, and the process of switches from type i to type j as a Poisson process with arrival rate $o_i r_i p_{ij}$, by the law of large numbers we model these aggregate stochastic process as a deterministic flow, and this means that the probability that an i-officer, $i = \{h, d\}$ will review his own strategy will be denoted by $r_i(O)$.

From these considerations it follows that:

- The outflow from the $i-$types' officials is:

$$\sum_{j \neq i} O_i r_i(\cdot) p_{ij}(\cdot).$$

- While the inflow is:

$$\sum_{j \neq i} O_j r_j(\cdot) p_{ji}(\cdot).$$

Being $O = (O_h, O_d)$ the profile distribution of officials' behavior, we apply the behavioral rule according to which a reviewing official who decides to change her current strategy takes into consideration imitating a strategy which performs better than her own current strategy. With the use of behavioral rules we can define an evolutionary dynamic as an inflow-outflow model. That is, rearranging terms, we get the system of differential equations characterising the dynamic flow of officials:

[8] This is the " behavioral rule with inertia" (see Bjornerstedt and Weibull, 1996; Weibull, 1995 and Schlag, 1998; 1999) that allows an agent to reconsider her action with probability $r \in (0,1)$ each round.

$$\dot{O}_h = r_d p_{dh} O_d - r_h p_{hd} O_h$$
$$\dot{O}_d = -\dot{O}_h, \tag{4}$$

meaning that the inflow of honest officers is given by the difference between the number of dishonest officers who decide to become honest ($r_d p_{dh} O_d$) and the number of honest officers who decide to stay honest ($r_h p_{hd} O_h$)

Assume that $r_i, i = d, h$ is population specific and is linear and decreasing in the level of the expected utility, i.e.

$$r_i = \alpha - \beta E(\cdot), \tag{5}$$

where $\alpha > 0$, $\beta \geq 0$ and $\frac{\alpha}{\beta} \geq E(\cdot)$ assure that $r_i(\cdot) \in [0,1]$. The parameter α is interpreted as the degree of dissatisfaction for following a behavior $i = \{h, d\}$ and β measures the weight of the payoff on the probability to be a reviewer. As long as the expected payoff level of the $i-$official, $E(\cdot)$ increases, her average reviewing rate r_i will decrease.

Reviewing officers evaluate their current strategy and decide to imitate only the successful one. Therefore, by the above considerations, the system (4) can be written as:

$$\dot{O}_h = -O_h\left[(\alpha - \beta E(O_h))p_{hd} - (\alpha - \beta E(O_d))p_{dh}\right] + (\alpha - \beta E(O_d))p_{dh}$$
$$\dot{O}_{dh} = -\dot{O}_h, \tag{6}$$

The share f_n of non-polluting firms is a constant number at any period of time t, and gross-payoffs π_i, salaries and effort (W, e) are given. Then $E(O_h)$ and $E(O_d)$ are constant too. Defining by $A = \left[(\alpha - \beta E(O_h))p_{hd} - (\alpha - \beta E(O_d))p_{dh}\right]$ and $B = (\alpha - \beta E(O_d))p_{dh}$, the solution of the differential equation (6) is

$$O_h(t) = \left(O_h(0) - \frac{B}{A}\right)\exp(-At) + \frac{B}{A}. \tag{7}$$

where $O_h(0)$ is the share of $h-$type officials at time $t = 0$ and,

$$\frac{B}{A} = \frac{(\alpha - \beta E(O_d))p_{dh}}{(\alpha - \beta(W - e))p_{hd} - (\alpha - \beta E(O_d))p_{dh}}. \tag{8}$$

Consider that officials copy successful behaviors according to a payoff-monotonic updating. An evaluation rule that seems fairly natural in a context of simple imitation, is the “ positive differences rule” , whereby a strategy is evaluated according to the differences in payoffs observed in the reference group (see J. Apesteguia et al., 2007). That is, each $i-$official changes her strategy if and only if $E(O_i) < E(O_j)$, $\forall i \neq j = \{h, dh\}$. Note that, there exists a threshold value $\bar{f}_n$ such that:

$$p_{ij}(x) \equiv \begin{cases} =1 & if \;\; E(O_j) - E(O_i) > 0. \\ <1 & otherwise. \end{cases}$$

Solution 1 *Officials follow an imitative behavior (equation (6)), then by the equation (7) when* $O_h(t)_{\lim t \to \infty} = \frac{B}{A}$ *the distribution of firms’ type* $F = (f_n, 1 - f_n) \in \Delta^F$ *determines the evolution path of corruption such that there exists a threshold value:*

$$\bar{f}_n = \frac{R + r_d - PM}{R - r_d},$$

and then it happens that:

the share of honest officials converges to one ($\frac{B}{A} = 1$) if $f_n > \bar{f}_n$ (no-corruption),

the share of honest officials converges to zero if $f_n < \bar{f}_n$ ($\frac{B}{A} = 0$) (all corrupt).

There is a mixed strategy equilibrium where the share of honest officials is given by:

$$\frac{B}{A} = \frac{(\alpha - \beta E(O_d))p_{dh}}{(\alpha - \beta(W - e))p_{hd} - (\alpha - \beta E(O_d))p_{dh}} \in (0, 1).$$

Therefore, as the share of non-polluting firms becomes larger, the share of honest officials increases at a rate depending on the reviewing rate r_i .

How to make corruptive behavior disappear? It is simple if we consider an high probability to detect corrupt behavior, i.e. $P=1$ and the particular case of $R=\theta F$, $r_d=\theta W$. Then, to eliminate corruptive behavior, it must be the case that the share of non-polluting firms, f_n^*, is larger than $\hat{f}_n^*$, that is to say

$$f_n^* > \hat{f}_n^* = \frac{\theta F - M}{\theta(F-W)+M}.$$

This implies that the dishonest official should be punished with a very high fine M.

4 The payoff-maximizing actions of the firms

Recall that we have a mixed model where firms are profit maximizers and the officers are imitators. The chosen behavior by firms depends on the expected payoff associated with each of the possible strategies, to be polluting or not. Of course a polluting firm can switch to being non-polluting, and in this case the accumulated waste is assumed to be transported quickly and efficiently to the nearby garbage collector, so we do not study this fact. However potentially switches are allowed because the proportion of honest officers might be different at different points in time.

Consider that firms maximize intertemporal profits in a market on which they face the next assumptions:

1. A demand structure that gives to each firm sales revenues $g(x(t))$ where $x(t)$ represents the capacity[9] of production at time t, and $x(t)>0$. We consider that g is a differentiable $C^2(0,\infty)$ function such that: $g(0)=0,\ g\geq 0,\ g^{'}>0$ and $g^{''}<0$.

2. The capacity of production is a differentiable function $x: R_+ \to R$ for each time t. Capacity is finite and bounded, so at every time $t \leq 0$, $x(t)\in\left[0,x^{'}\right]$.

3. A fraction $(1-u(t))g(x(t))$ of the revenues is consumed, and what is left,

[9] For simplicity, we assume that the production's capacity is always fully exploited, so $x(t)$ can be interpreted as both capacity and level of production.

$u(t)g(x(t))$, is invested in new production capacity at a price $1/a > 0$.

4. An official inspects the firm at the end of the planning period T.

5. Each firm can buy an initial production capacity at the unitary price c and can sell it at price w at the end of the planning period T.

6. The production process generates pollution. Let $z(t)$ be the accumulated waste at time t. The istantaneous variation of waste $\dot{z}(t)$ is proportional to the used production capacity (level of production), so $\dot{z}(t) = bx(t)$, with $b > 0$.

7. Nonpolluting firms have an environmental cost of production (or cost to keep clean the environment) that is proportional to the waste accumulated until time t, $C(t) = \int_0^t bz(t)e^{-r\tau}d\tau$, $b \in (0,1)$. So, at the end of the period T, $C(c) = C(T) = \int_0^T bz(t)e^{-rt}dt$. The intertemporal discount rate is constant and equal to $r > 0$.

Therefore the profit of a non-polluting firm, $\pi_n(T)$, in period T, is given by the following maximization program:

$$\begin{cases} \pi_n(T) - C(T) = \max\limits_{u(t)\in 0,1]} \int_0^T [(1-u(t))g(x(t)) - bz(t)]e^{-rt}dt - cx(0) + wx(T)e^{-rT} \\ \dot{x}(t) = au(t)g(x(t)), x(0) \quad free, \ x(T) \quad free \\ \dot{z}(t) = bx(t), \ z(0) = z_0, \ z(T) \ free. \end{cases} \tag{9}$$

Since a polluting firm does not pay for the environmental cost of production, its maximization program is given by:

$$\begin{cases} \pi_p(T) = \max\limits_{u(t)\in 0,1]} \int_0^T [(1-u(t))g(x(t))]e^{-rt}dt - cx(0) + wx(T)e^{-rT}. \\ \dot{x}(t) = au(t)g(x(t)),\ x(0) \quad free,\ \ x(T) \quad free. \end{cases} \tag{10}$$

Notice that the fraction of the sales, $(1-u(t))g(x(t))$, is consumed and the rest, $u(t)g(x(t))$, is invested in making a new production bought at a price $1/a$. Without loss of generality, let us consider for the cases (9) and (10) the following inequality:

$$c > 1/a > w. \tag{11}$$

Because the initial capacity is financed by a loan having an interest rate higher than r while the gradual increase in capacity $ug(x(t))$ is paid for immediately. To ensures that it pays to invest, i.e. $x(0) > 0$, we consider that:

$$r^{-1}(1-e^{-rT})(g'(x(0))) > c - we^{rT}. \tag{12}$$

Hence, the Hamiltonian for the problem (??), with $p_0 = 1$,[10] is:

$$H(x,z,u,p_1,p_2,t) = [(1-u)g(x) - bz]e^{-rt} + p_1 aug(x) + p_2 bx. \tag{13}$$

and the candidate for optimality $(x^*(t), z^*(t), u^*(t))$ satisfies:

$$u^*(t) \ maximizes \ u(t)g(x)\left[p_1 a - e^{-rt}\right].$$

So

$$u^*(t) = \begin{cases} 1 & if \ \ p_1(t) > (1/a)e^{-rt} \\ 0 & if \ \ p_1(t) < (1/a)e^{-rt} \\ \in (0,1) & if \ \ p_1(t) = (1/a)e^{-rt} \end{cases} \tag{14}$$

and,

$$\dot{p}_1(t) = -\frac{\partial H^*}{\partial x} = -\left[(1-u^*(t))e^{-rt} + p_1(t)au^*(t)g'^*(x^*(t))\right] \tag{15}$$

[10] Notice that, if $p_0 = 0$ then: $\dot{p}_2 = -\frac{\partial H}{\partial z} = 0 \Rightarrow p_2(t)$ is a constant. From the transversality condition it follows that $p_2(t) \equiv 0$. From the maximum principle is necessary that: $(p_0, p_1(t), p_2(t)) \neq (0,0,0)\ \forall\, t$, then $p_1(T) \neq 0$, but this contradicts the transversality condition.

Taking into account the value of $u^*(t)$ it follows that:

$$\dot{p}_1 = -\left[\max\left\{e^{-rt}, ap_1(t)\right\}\right]g'^*(x^*(t)), \tag{16}$$

and then $\dot{p}_1(t) < 0.$ So $p_1(t)$ is a decreasing function.

From the necessary conditions it follows that:

$$p_1(0) = c \ and \ \ p_1(T) = we^{-rT}.$$

The next proposition states a crucial result from the above considerations of this section.

Proposition 1 *There is a moment* $t = T^* > 0$ *such that the equality* $p_1(T^*) = (1/a)e^{-rT^*}$ *follows. The difference between the benefits of the polluting and the nonpolluting firms increases with* T^**.*

Proof. Denote by $\phi(t) = \frac{1}{a}e^{-rt}$. Since $c > 1/a > w$, it follows that $p_1(0) > \phi(0)$ and $p_1(T) = we^{-rT} < \frac{1}{a}e^{-rT}$ being $p_1(t)$ strictly decreasing, then there exists at lest one $T^* > 0$ such that $p_1(T^*) = \frac{1}{a}e^{-rT^*}$. We need to prove that this solution is unique. Thus, in time $t = T^*$ the equality $p_1(T^*) = (1/a)e^{-rT^*}$ follows.

Hence:

$$x^*(t) = \begin{cases} a \ solution \ of \ \ \dot{x}(t) = ag\left(x(t)\right) & \forall \ t \in \left[0, T^*\right] \\ \\ x(T^*) & \forall \ t \in \left[T^*, T\right] \end{cases} \tag{17}$$

$$\dot{p}_2(t) = -\frac{\partial H^*}{\partial z} = be^{-rt} \rightarrow p_2(t) = -\frac{b}{r}e^{-rt} + p_2 \ where \ p_2 \ is \ a \ constant.$$

Moreover, note that the differences in profits between the polluting and nonpolluting firms is given by:

$$\pi_p(T) - \pi_n(T) = b\left[\int_0^{T^*} x(t)e^{-rt}dt + x(T^*)[T - T^*]e^{-r(T-T^*)}\right], \tag{18}$$

Since the expression that appears in the integrand is always positive, the higher the value of T^* the greater the difference between these two benefits.

The same result holds for the expected values, according with equations (1) and (2):

$$E(\pi_p)-E(_n)=b\left[\int_0^{T^*}x(t)e^{-rt}dt-\frac{x(T^*)}{r}\left(e^{-rT}-e^{-rT^*})\right)\right]-R-r_d+[-F+R+r_d]O_h. \quad (19)$$

To get an analytical solution we consider the usual case $g(x)=x^{\frac{1}{2}}$. From equation (16) it follows that:

$$\dot{x}(t)=a(x(t))^{\frac{1}{2}},\forall t\leq T^*.$$

So,

$$x^*(t)=\begin{cases}\frac{1}{4}a^2t^2+x(0) & \forall t\in 0,T^*] \\ \frac{1}{4}a^2(T^*)^2+x(0) & \forall\, t\in\left[T^*,T\right]\end{cases} \quad (20)$$

From equation (16), $\forall\ t\leq T^*$ we get that:

$$\dot{p}_1=ap_1(t)\frac{1}{2}\left(x^*(t)\right)^{-\frac{1}{2}}, \quad (21)$$

Substituting (20) in th above equation we obtain:

$$\dot{p}_1=-ap_1\frac{1}{2}\left[\frac{1}{4}a^2t^2+x(0)\right]^{-\frac{1}{2}}, \quad (22)$$

and integrating, we obtain that:

$$\ln p_1=-\arctan\frac{a}{2\sqrt{x_0}}t+C, \quad (23)$$

where C is a constant on integration. Taking exponential on both sides of (23) and $p_1(0)=c$, hence the expression:

$$p_1(t)=ce^{-\arctan\frac{a}{2\sqrt{x_0}}t}$$

holds.

The difference in benefits between a polluting and nonpolluting firm is given by: $\pi_p(T)-\pi_n(T)=$

$$\begin{cases} \int_0^{T^*} [\frac{1}{4}a^2t^2 + x(0)]e^{-rt}dt & \text{if } T < T^* \\ \int_0^{T^*} [\frac{1}{4}a^2t^2 + x(0)]e^{-rt}dt - [\frac{1}{4}a^2(T^*)^2 + x(0)][T - T^*][e^{-rT} - e^{-rT^*}] & \text{if } T > T^*. \end{cases} \tag{24}$$

Notice that such a difference between the benefits of the polluting and nonpolluting firm increases with T^*.

4.1 The rate of pollution and the discount factor

According with the equation $\dot{z} = bx^*(t)$ the instantaneous velocity of pollution accumulation, (or the rate at which pollution accumulates) increases with time $t \le T^*$ and after this moment it is a constant: $\dot{z} = bx(T^*), \forall\, t > T^*$. Then, the period during which the instantaneous velocity of contamination grows, it is an increasing function of the discount rate. These facts are summarized in the following proposition:

Proposition 2 *If there exists a solution T^* for the equation $\chi(T, r)) = p_1(T) - \frac{1}{a}e^{-rT} = 0$, then there exists a neighborhood V_{r^*} of r^* such that for each $r \in V_{r^*}$ there exists only one optimal time $T^*(r)$ such that the instantaneous velocity at which the pollution is created increases until $t = T^*$, after that time the rate of pollution does not increase. This optimal time increases with the discount factor r.*

Proof. Consider the function,

$$\chi(T, r)) = p_1(T) - \frac{1}{a}e^{-rT}$$

Since, for a given $r^* > 0$ there exists T^* such that $\chi(T^*, r^*) = 0$. From the implicit function theorem there exists a continuous function $T^*(r)$ such that $\chi(T(r), r) = 0$. Now using the derivative of the implicit function, it follows that:

$$\frac{dT}{dr} = -\frac{\partial\chi/\partial r}{\partial\chi/\partial T} = -\frac{\frac{1}{a}Te^{-rT}}{\dot{p}_1 + \frac{1}{a}re^{-rt}}$$

Since $p_1 < -\frac{1}{a} re^{-rt}$ then $\frac{dT}{dr} > 0$. This means that as low is the discount rate, lower is the optimal time until the rate of pollution increase. See Figure 1.

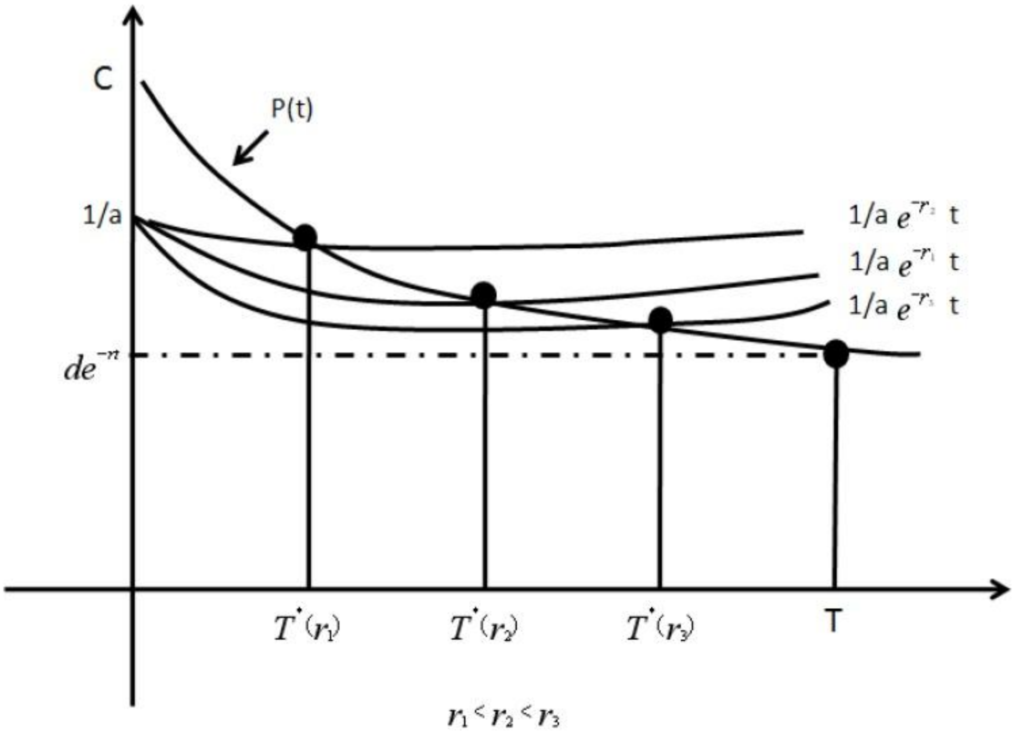

Figure 1. Optimal time *T**as a function of the discount rate r.

Finally, for the second case (program (10)) we have a similar situation except for $b = 0$. The Hamiltonian to this problem (10) with $p_0 = 1$, is given by:

$$H(x,u,p_1,t) = (1-u)g(x)e^{-rt} + p_1 auf(x) \tag{25}$$

and the candidate for optimality $(x^*(t), u^*(t))$ verifies the similar conditions of the previous one. The maximized Hamiltonian $H^* = \max\{e^{-rt}, ap_1(t)\}g(x)$ is strictly concave on x if $g(x)$ is a strictly concave function, and so (x^*, u^*) is a solution to this problem.

For the particular case where $g(x) = x^{\frac{1}{2}}$, we can get the equation:

$$ce^{-\arctan\frac{a}{2\sqrt{x_0}}T} = \frac{1}{a}e^{-rT} \tag{26}$$

and it follows that T^* is a solution of the equation:

$$-\arctan\frac{a}{2\sqrt{x_0}}T + rT + \ln ac = 0 \qquad (27)$$

Choosing the parameters of the model verifying the conditions: (12) and (11) then the solution T^* for this equation exists.

4.2 A threshold value for the dynamics of firms

In this section we show that there exists a threshold value such that once the share of honest officials exceeds this value, then a process in which polluting firms prefer to become nonpolluting begins, and the current nonpolluting firms remain non-polluting.

Recall that firms maximize their expected profits, so they prefer to be polluting if:

$$E(f_p) - E(f_n) > 0. \qquad (28)$$

and this happens if the share of honest officials is smaller, i.e.

$$O_h < \frac{b\left[\int_0^{T^*} x(t)e^{-rt}dt - \frac{x(T^*)}{r}\left(e^{-rT^*} - e^{-rT}\right)\right] - R - r_d}{F - R - r_d}. \qquad (29)$$

Recall that the profile distribution of firms' type is given by $\left(f_p(t), f_n(t)\right) \in \Delta^F$ in time t. Consider that at the end of each period, firms choose their behavior for the next one. Assume that at each time t, firms know the officials' distribution $O(t)$, (i.e. they know the probability to be inspected by an honest or a dishonest official) then the dynamics of the share of firms is given by the next law of motion:

$$\begin{cases} \dot{f}_n = \left[E(f_n) - E(f_p)\right]f_p \\ \\ \dot{f}_p = -\dot{f}_n. \end{cases} \qquad (30)$$

If $E(f_n) - E(f_p) > 0$ then the share of non-polluting firms increases.

Let us introduce the function: $O_h^T : R_+ \to R$ defined by:

$$O_h^T(r) = \frac{b\left[\int_0^{T^*} x(t)e^{-rt}dt - \frac{x(T^*)}{r}\left(e^{-rT^*} - e^{-rT}\right)\right] - R - r_d}{F - R - r_d}. \qquad (31)$$

This function defines (for the discount rate r) a threshold value of honest officials $O_h^T(r)$

such that if $O_h^T(r) < O_h$, then the share of non-polluting firms increases. The next proposition summarizes the above consideration.

The following proposition shows that the threshold value is an increasing function of r. It is straightforward the intuition behind this proposition: if firms care less about the future, more should be be the society's efforts to prevent pollution.

Proposition 3 *The threshold value,* $O_h^T(r)$, *is increasing function with* r.

Proof. To prove this theorem let us consider the auxiliary function:

$$\phi(r) = -\int_0^{T^*(r)} tx(t)e^{-rt}dt + x(T^*(r))\int_{T^*(r)}^{T} e^{-rt}dt.$$

From proposition (2) we know that $T^*(r)$ is an increasing function of r, that $(T^*)'(r) > 0$ and that $0 < T^*(r) < T$. It follows that $\phi'(r) > \mathbf{0}$, so this function is increasing on r, i.e.

$$\phi'(r) = -\int_0^{T^*(r)} tx(t)e^{-rt}dt - x(T^*(r))e^{-rT^*(r)}T^{*'}(r) + \dot{x}(T^*(r))T^{*'}(r)\int_{T^*}^{T} e^{-rt}dt +$$

$$+ x(T^*(r))e^{-rT^*(r)}T^{*'}(r) - x(T^*(r))\int_{T^*(r)}^{T} te^{-rt}dt =$$

$$= -\left[\int_0^{T^*(r)} tx(t)e^{-rt}dt + x(T^*(r))\int_{T^*(r)}^{T} te^{-rt}dt\right] > \mathbf{0}$$

The second equality is a consequence of the fact that $x(t) = x(T^*), \forall\, t \in T^*, T]$ hence $\dot{x}(t) = 0$

Two important insights are:

1. The intuition of proposition (3) is given by the fact that when future does not matter at all, the discount rate is high and the current environmental cost of production is low, since we do not care about environment. When we care about future, then we may clean the current environment such that the environmental cost of production is higher. Then if the current cost of cleaning is lower and honest officials are few, $E(f_p) < E(f_n)$.

2. Note that if:

$$\int_0^{T^*} x(t)e^{-rt}dt - \frac{x(T^*)}{r}\left(e^{-rT} - e^{-rT^*})\right) > \frac{F}{b}, \quad (32)$$

then independently of the officials' distribution, firms prefer to be polluting. That is contrary to intuition, because if the value $\int_0^{T^*} x(t)e^{-rt}dt - \frac{x(T^*)}{r}\left(e^{-rT} - e^{-rT^*}\right)$ is higher than the fine, a policy of raising the fine F may not be efficient. This may explain why increasing the fine makes the option of offering a bribe more attractive, thus inducing more corruption. The right value of F should be fixed (exogenously to our model) to the correct value that the society worries about the future, so is reducing the size of the bribes and implement a policy for increasing the probability of the time when there is compliance and no corruption.

Therefore it is important to develop a policy aimed at creating awareness about the future, so as to diminish the value of the discount rate.

5 Conclusion

This paper develops a model of corruption based on imitation, in an environmental policy context where (potentially corrupt) officers report pollution produced by firms. Officers might be honest or dishonest while firms may be polluting or not.

We identify several equilibria in the static game, which are confirmed by extending such a game in an evolutionary setting where officials' imitate the others' strategy and firms maximise profits. Equilibria range from stable corruption to honesty depending on the parameters of the model (i.e. fines, bribes and environmental damages as well as the firms' discount rate).

When firms care about future (i.e. a low discount rate) and officials are honest then we get the good outcome implying an economy without corruption, but the worst scenarios occurs when all the firms are briber and officials are dishonest.

To encourage an honest behavior, that is to say, a situation where firms prefer to be clean and officers prefer to be honest, bribes' size must be reduced, fines (M) must be increased, and P, the probability to detect dishonest behaviours performed by an officer must be greater, that is to say, the government must invest in increasing its effectiveness in detecting corrupt officers.

When this effectiveness is increased by means of the firm which receives an unfair report could be the object of future research, since the hypothesis that a firm prefers not to appeal to a court because of the cost and the long bureaucratic process may seem not completely realistic.

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