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# Systemic Banks and the Lender of Last Resort

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**Systemic banks and the lender of last resort**[1]

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## Abstract

We propose a model whereby systemic and non-systemic banks are exposed to liquidity shortfalls such that a lender of last resort policy is required. We analyze the optimal allocation of lender of last resort responsibilities and find that it is socially optimal to move responsibilities from the central bank to an unconditional bailout rule when the shortfall is large enough. The existence of systemic banks provides a rationale for the central bank to act as lender of last resort for non-systemic banks in a larger range of their liquidity shortfalls. However, the impact of considering systemic risk on the optimal allocation of the lender of last resort responsibilities for systemic banks is ambiguous.

*Keywords:* Systemic banks, systemic risk, lender of last resort policy.

## Resumen

En este artículo se utiliza un modelo para analizar la distribución óptima de la responsabilidad de prestamista de última instancia cuando bancos sistémicamente importantes coexisten con bancos no sistémicos. Es socialmente óptimo que un banco central actúe como prestamista de última instancia para problemas pequeños de liquidez, y que esa responsabilidad sea sustituída por una regla de soporte irrestricto cuando los problemas de liquidez sobrepasan un umbral predefinido. La existencia de bancos sistémicamente importantes provee una razón para que el banco central actúen como prestamista de última instancia de bancos no sistémicos en un mayor rango de sus problemas de liquidez. Sin embargo, el impacto de considerar el riesgo sistémico sobre la distribución de responsabilidades de prestamista de útlima instancia para con bancos sistémicamente importantes es ambigua.

*Palabras clave*: bancos sistémicos, riesgo sistémico, prestamista de última instancia.

*JEL:* G21, G28

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# 1 Introduction

A series of observations from the recent financial crisis, although shared by many other crises in the past, motivate this article. First, interbank markets may collapse so that even solvent banks are unable to access funding to finance their operations.[4] Second, large or highly interconnected (i.e. systemic) financial institutions were at the center of the fragility of the financial system and their problems rapidly spread over non-systemic financial institutions.[5] Third, the policy response to the crisis involve the provision of large amounts of emergency liquidity assistance and several structural reforms of the regulatory framework. Many times during the Subprime crisis governments and central banks supported financial institutions with liquidity independently of their solvency condition. The rational for providing this liquidity support was to stabilize the financial system and to prevent further contagion effects.[6] Structural reforms of the regulatory framework aim to enhance the resilience of financial institutions and to promote the resolution of distressed systemically important financial institutions in an orderly manner.[7] Although it proved to be a very important issue during the crisis, the implications of systemically important financial institutions for the design of the lender of last resort policy to provide funding to banks in case external sources of liquidity dry up has not received much attention among policymakers nor among academics. This paper aims to contribute towards filling this gap in the literature.

We present a formal model which is inspired by Repullo (2000) . In the model,

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[4] Gorton and Metrick(2011) show that during the Subprime crisis in 2007-2008 crisis the repo market collapsed. Increasing haircuts of bilateral repo transactions combined with declining asset values reduced the funding capacity of the banking sector. Copeland et al. (2011) argue that also tri-party repo markets dried up because the amount of funding decreased sharply.Acharya and Merrouche (2010) provide evidence for the liquidity hording and the effect on overnight interbank rates during the subprime crisis. See Brunnermeier (2008) and Mishkin (2010) for a description of the evolution of the financial crisis and its main events.

[5]Acharya et al. (2010) measure the systemic risk of individual financial institution during and after the Subprime crisis. They show that size and interconnectedness are good determinants of the contributions of individual financial institutions to systemic risk. In particular, financial institutions like Lehman Brothers, Merril Lynch, Bear Stearns or AIG impose a large systemic risk for the US financial system.

[6] For example in Europe liquidity interventions by governments add up to around 30% of the its GDP. See, for instance, http://europa.eu/legislation_summaries/internal_market/single_market_services/financial_services_banking/mi0062_en.htm.

[7] The Basel Committee on Banking Supervision and the Financial Stability Board published on the 4th of November 2011 press releases presenting specific requirements for globally systemically important banks: http://www.bis.org/press/p111104.htm and http://www.financialstabilityboard.org/press/pr_111104cc.pdf. The reforms include new capital requirements to systemically important institutions, the elaboration in advance of resolution plans for this type of institutions, and the enactment of more efficient supervision.

external sources of funding (e.g. the interbank market) are not available for illiquid, although maybe solvent, banks so that only an emergency liquidity loan from a lender of last resort (LLR) can ensure the bank's continuation. Banks engage in maturity transformation by investing demand deposits into risky, illiquid, long-term assets. A liquidity shortfall occurs at an intermediate date, which is modeled as a random withdrawal of demand deposits.[8] We analyze the implications of the existence of systemically important banks for the design of the lender of last resort policy.[9] In our model a systemic bank coexists with a non-systemic bank. The failure of the systemic bank may hurt the return of the non-systemic bank but not vice versa. We find that it is first-best socially optimal to provide emergency liquidity assistance to banks with assets of high quality, while low-asset-quality banks should be closed down. However, the policy maker cannot implement this first-best policy because it cannot verify the quality of a bank's assets and therefore its solvency condition. Hence, it either delegates the decision to an agency which observes the solvency signals through supervision or applies a prefixed policy rule. The problem of the policy maker is to announce ex ante the allocation of the lender of last resort responsibilities in order to maximize expected social welfare. We assume that the liquidity shortfall is verifiable. Hence, the policy maker may allocate responsibilities conditional on the size of the required liquidity assistance. We concentrate the analysis on two cases: one in which the central bank is the lender of last resort and another in which an unconditional bailout rule is applied. In the first case it is the central banker who makes the decision of providing an emergency loan to the illiquid bank. In the second case the central banker is instructed to provide an emergency loan regardless of the solvency condition of the illiquid bank. In an extension we show that considering other candidates to act as lender of last resort (e.g. the deposit insurance corporation) does not improve the optimal allocation of responsibilities that is obtained by considering these two alternatives.

In a benchmark case with only one type of bank (e.g. non-systemic banks) we show that it is second-best socially optimal to share lender of last resort responsibilities between the central bank and the unconditional bailout rule. The second-best optimal allocation of the lender of last resort responsibilities consists of two intervals. For banks showing small

[8] The withdrawal of deposits is only modeled in reduced form because the study of the incentives of depositors is outside of the scope of this paper.

[9] The recent crisis provide several examples of interbank and money markets closure, yet several theoretical papers, see for example Allen et al (2009), Flannery (1996), Freixas and Jorge (2008), Rochet and vives (2004) argue that market imperfections may imply that interbank markets only achieve a second-best allocation and that public interventions by a lender of last resort may improve social welfare.

liquidity shortfalls the central banker should act as lender of last resort. For large liquidity shortfalls the unconditional bailout rule should be applied. The rationale for this allocation is as follows. The central bank is concerned about its expected utility from the lender of last resort activities because it incurs monetary losses and political costs when a bank fails. As a result, the central banker, in providing an emergency loan, requires that the assets quality of illiquid banks increases in proportion to the size of their liquidity shortfalls. For large enough liquidity shortfalls the solvency requirement of the central banker widely exceeds the first best social optimal, i.e. the central banker closes down too many banks. Hence, the social planner prefers to support illiquid banks regardless of their solvency condition, which is implemented through the application of an unconditional bailout rule.

When we consider that systemic banks coexist with non-systemic banks the qualitative results are as in the benchmark case: the central bank should act as lender of last resort for small liquidity shortfalls and the unconditional bailout rule should be applied when shortfalls exceed a certain threshold. However, the existence of systemic banks provides a rationale for the central bank to act as lender of last resort in a larger range of liquidity shortfalls for the non-systemic bank. The result can be explained in the following way. The optimal threshold for overriding the central banker's lending decision through the use of the unconditional bailout rule for the non-systemic bank depends on the state of the systemic bank. Since the failure of the systemic bank decreases the expected return of the non-systemic bank the minimum solvency requirement to support the illiquid, non-systemic bank is from a first-best point of view higher than in the benchmark case. Hence, the central banker's lending decision is closer to the first-best over a larger set of liquidity shortfall. As a result, the central banker should be the lender of last resort for the non-systemic bank on a larger range of its liquidity shortfalls.

However, we are not able to prove a non-ambiguous effect of the existence of systemic risk on the optimal allocation of responsibilities for the systemic bank because there are two counteracting effects. One the one hand, the existence of systemic risk implies that the social planner will be biased towards forbearance with the systemic bank. Other things equal, this implies a more frequent use of the unconditional bailout rule for systemic banks. On the other hand, the central banker itself will be less strict because it anticipates higher expected losses in its granting of last resort loans to the non-systemic bank when the systemic one fails. Everything else being constant this implies that the central banker should receive more responsibilities as lender of last resort for the systemic bank. The final outcome depends on the relative strengths of these two effects.

The rest of the paper is organized as follows. Section 2 provides a review of the related literature. In Section 3 we introduce the basic model. In Section 4 we present the benchmark case, in which only on type of bank (e.g. non-systemic) exists, for further references. We introduce systemic risk into the model in Section 5 where we show the main findings of this paper. In section 6 we extend the set of candidates to act as lender of last resort by considering the deposit insurance corporation in addition to the central bank and the unconditional bailout rule. In Section 7 we offer some final remarks.

# 2 Related literature

Our model builds on the previous literature on the lender of last resort policy and on the institutional allocation of lender of last resort responsibilities, borrowing extensively from its insights.[10] Closely related papers are by Espinosa et al. (2011), Ponce (2010) and Repullo (2000). Repullofirst considers the question of the optimal institutional allocation of lender of last resort responsibilities in the incomplete contracts framework of Dewatripont and Tirole (1994). Espinosa et al. (2011) extend Repullo´s (2000) model by introducing systemic risk and analyze whether or not a unified regulator, i.e. the lender of last resort combined with the deposit insurance in a single agency, is superior to an architecture with separated agencies. We build on their insights and analyze the optimal institutional allocation of lender of last resort responsibilities between the central banker and the unconditional bailout rule. In so doing we are extending the analysis by Ponce(2010) in order to consider systemic risk.

The optimal institutional allocation of lender of last resort responsibilities was

[10] The concept of the lender of last resort can be traced back to the work by Bagehot (1873) and Thornton (1802). They state that the central bank should act as the lender of last resort lending to solvent banks, at a penalty rate and requiring good collateral. Goodfriend and King (1988) argue that the existence of an interbank market makes the liquidity provision to individual banks unnecessary. Goodhart (1999) points out that it is difficult for the central bank to distinguish between solvent and insolvent banks, and that the lender of last resort might not be better informed than the market. Therefore, the lender of last resort allocation should be inferior to the market allocation. Castiglionesi and Wagner (2012) show that under some conditions penalty rates increase banks' moral hazard. However, Rochet (2004) provides a rationale for a lender of last resort in a framework with sophisticated interbank markets. Flannery (1996), Freixas et al. (2000), Rochet and Vives (2004) focus on coordination failures in interbank markets and provide further rationale for lender of last resort interventions. In these papers the existence of a lender of last resort can assure market participants and prevent inefficient closure of solvent banks. Moreover, as in Acharya and Yorulmazer (2008) emergency liquidity loans provide the surviving banks with necessary liquidity to acquire the illiquid banks' assets and avoid efficiency losses due to misallocation of assets. Overall, Bagehot's (1873) doctrine is widely accepted among academics and policymakers.

initially studied by Repullo (2000). In his model a lender of last resort decides about the provision of emergency liquidity assistance to banks that are suffering from a liquidity shortfall. The banks' solvency is private information so that only the lender of last resort is given the authority to evaluate banks and receive a perfect but nonverifiable signal about their solvency. Two agencies may act as a lender of last resort: the central bank and the deposit insurance corporation. Both agencies have the objective to maximize their expected final wealth. But they differ in their mandates so that their individual lending decisions as a lender of last resort do not coincide. The deposit insurance corporation has the obligation to compensate depositors in case of a bank's failure. When refusing the emergency loan it can liquidate banks in trouble, realize the liquidation value and limit its losses from the lender of last resort activities. For this reason the deposit insurance corporation is biased towards prompt liquidation in order to maximize the liquidation value of the bank. The central bank's engagement is restricted to the emergency loan. It grants the emergency loan conditional on the bank's solvency signal. Repullo shows that the deposit insurance corporation is always tougher than socially optimal. The central bank on the contrary is too soft for small liquidity shocks, but too restrictive for large liquidity shortfalls. In Repullo's framework the second-best optimal allocation involves both agencies. The central bank should be in charge of the lender of last resort responsibilities for small liquidity shortfalls while the deposit insurance should decide about the liquidity assistance for larger liquidity shocks.

Kahn and Santos (2005, 2006) use Repullo´s (2000) framework to study the merits of centralization of lender of last resort responsibilities and the deposit insurance function.They find that centralization induces more forbearance for large liquidity shocks and leads to inefficient investment into the risky asset. Keeping the functions separated causes softer lending decisions for small liquidity shortfalls. By considering the existence of informational frictions about the bank's solvency and liquidity shock they show that the central bank does not have an incentive to share its private information.

Ponce (2010) extends Repullo´s (2000) framework by introducing an unconditional bailout rule meaning that an emergency loan will be provided to the bank in trouble regardless of the bank's solvency. He shows that the second-best optimal allocation consists of the application of the unconditional bailout rule for large liquidity shocks and the allocation of the lender of last resort responsibility to the central banker for small liquidity shocks. Since Banks may be able to manipulate the size of the liquidity shortfall, the application of the unconditional bailout rule should be complemented by a punishment to the banker in order to deter him from manipulating the liquidity shortfall. Moreover, he shows that first-best allocation can be achieved with an appropriate compensation scheme for the central banker.

Espinosa et al. (2011) introduce a systemic bank into Repullo´s (2000) model. As in Kahn and Santos (2005,2006), their objective is to study the effect of centralization of regulatory arrangements on the incentives of regulatory agencies to exert forbearance and to share information. They show that, under an expanded mandate to explicitly oversee systemic risk, regulators would be more forbearing towards systemically important institutions and that regulators may have little incentive to share it with other regulators. They conclude that, under some conditions, an unified regulatory arrangement can reduce the degree of systemic risk vis-a-vis a multiple regulatory arrangement.

In this paper, we extend Ponce (2010) by considering that a systemic bank coexists with a non-systemic bank. As in Espinosa et al. (2011) we assume that the failure of the systemic bank leads to a lower expected return on the assets of the non-systemic bank, and that the failure of the latter does not have effects on the former.[11] Our objective is different: we study the implications of considering systemic risk for the optimal allocation of lender of last resort responsibilities.

# 3 The model

We propose a model inspired by Repullo (2000) where banks are funded entirely by demand deposit contracts. More precisely banks raise one unit of deposits at the beginning of their operations. We assume that deposits are fully insured by the deposit insurance and that they can be withdrawn either after the first or the second period of operation.

The banks invest their deposits into an illiquid risky asset which yields a random return $\tilde{R}$ for each unit invested after two periods. The asset can either succeed, $\tilde{R} = R$, or fail, $\tilde{R} = 0$. The asset is ex ante profitable: $E(\tilde{R}) > 1$. The entire bank can be liquidated at

[11] Espinosa et al. (2011) model this point by assuming that the failure of the systemic bank reduces the probability of success for the non-systemic bank. We model it differently: we assume that the failure of the systemic bank reduces the return of the non-systemic bank. While both assumptions allow capture of the externality that systemic banks may impose on non-systemic ones, our approach makes the algebra easier.

the intermediate date. The liquidation value is equal to $L \in (0,1)$.

As in  Espinosa et al. (2011) we consider two types of banks: a systemic bank (S) and a non-systemic one (N). A bank is considered as systemic if its failure has a negative effect on the non-systemic bank. We assume that the systemic impact reduces the return of the non-systemic bank's asset in the successful state to $\widetilde{R} = R - \gamma$. We differ in the modeling of the contagion effect from Espinosa et al. (2011), but our approach follows Rochet and Tirole (1996) where "*systemic risk refers to the propagation of an agent's economic distress to other agents linked to that agent through financial transactions*". From this point of view the systemic impact can be interpreted as losses from interbank or payment system claims against the systemic bank and is therefore related to the counterparty risk within a financial system. E.g. in interbank markets banks are connected through interbank lending in order to manage liquidity preferences. As a consequence of the systemic bank's collapse the non-systemic bank's asset e.g. a portfolio consisting of several assets including claims against the systemic bank yields a lower return. In such a framework  Freixas00 show that the failure of a systemic bank spills over to other financial institutions and can trigger liquidations of non-systemic banks.

A bank failure can occur because after the first period of operation a fraction $v \in (0,1)$ of banks' deposits are withdrawn. The sudden withdrawal of deposits can be interpreted as depositors' consumption preferences as in Diamond and Dybvig (1983). However, we do not model depositors' behavior in detail, because the focus of this paper is on the optimal allocation of lender of last resort responsibilities. The depositors' behavior is beyond the scope of this paper. We assume that the withdrawal behavior of depositors like queuing in front of banks during a bank run is publicly observable so that the liquidity shock $v$ is publicly verifiable. The liquidity shock $v$ corresponds to the realization of a random variable $\tilde{v}$ with a cumulative distribution $G$ with support in [0,1].

Since banks do not hold any liquid reserves and assets are completely illiquid, banks face bankruptcy if $v > 0$ unless the lender of last resort provides emergency liquidity assistance. A closure of a bank causes social costs of $c$. The social costs include, for example, bankruptcy costs and costs related to negative effects on the economy beyond the banking sector. We assume that the liquidity shocks of both banks are independent. This implies that we focus on individual liquidity situation and do not consider contagion effects of system liquidity crisis.

Additionally, there exists uncertainty about the success probability of the bank's asset in the model. Simultaneously with the liquidity shock $v_i$ a perfect but non-verifiable signal $u_i$ with $i \in S, N$ about the success probability of the bank's asset at maturity is realized. The signal is privately observed only by the agency assigned with the LLR responsibilities, because it has the authority to collect all necessary information and the ability to asses the quality of banks' assets by supervision in order to fulfill this task. The solvency signal is non-verifiable because it may be based on soft information obtained during asset quality assessment process. This assumption is decisive for the lender of last resort policy because ex ante allocation of responsibilities has to be conditional on the liquidity shortfall $v_i$.

The policy maker can allocate the lender of last resort responsibilities between the central banker and the unconditional bailout rule in order to maximize social welfare. In the public sector many agencies have multidimensional mandates including the achievement of the agencies' aims at reasonable cost. According to Tirole (1994) this does not prevent the policy maker designing a mechanism to motivate agencies if two concerns are considered. First, the quantification of some dimensions might be difficult. While the failure of a bank is publicly observable the decision making of the agency to ensure the stability of the financial system might be private information. For this reason the central banker has to bear political cost in case of a failure under his mandate. Second, due to the existence of multiplicity of dimensions the allocation of weights to the different dimensions is of concern. We incorporate Tirole´s (1994) basic ideas and follow Ponce (2010) by setting up the objective function for the central banker so that it cares about its financial wealth, net of incurred political cost from a bank failure:

$$U = I - \alpha 1_{\{failure\}} c, \tag{1}$$

where $I$ corresponds to the agency's net income, $1_{\{failure\}}$ is equal to one if the bank fails and zero otherwise, and $\alpha$ is the weight given to the political cost for the central banker in case of a bank's failure. Like Repullo (2000) and Ponce (2010) we assume that the political cost of a bank's failure for the central banker does not exceed the social cost ($\alpha < 1$). We argue that the central banker can only be blamed for a fraction of the social cost caused by a bank failure because the society will hold the central banker responsible for the realized social cost at most.[12]

The central banker's net income from the lender of last resort responsibilities is determined by its mandate. Its exposure corresponds to the amount of the emergency loan when it is engaged in liquidity provision. In case the troubled bank fails after being supported the central banker loses its emergency loan.

As in Ponce (2010) apart from allocating the responsibility to the central banker the policy maker can implement an unconditional bailout. In this case the central banker is instructed to provide liquidity to the troubled bank without any negative effect on its utility in case of default. In this case the central banker does not incur any political cost from a failure when the unconditional bailout rule is applied.

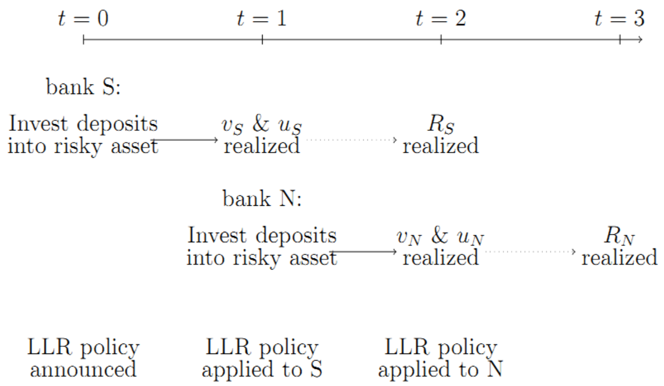


Figure 1: Timing of the model.

The timing of the model is summarized in figure 0 and will be explained in the following. For simplification but without loss of generality the systemic bank $S$ starts to operate at date 0 while the starting date of operation for the non-systemic bank $N$ is delayed to date 1. This sequential structure avoids the simultaneity of events and facilitates the analysis of the lender of last resort policies for both banks.

At date 0 the policy maker announces the lender of last resort policy for the systemic bank S and the non-systemic bank N. Bank S raises one unit of deposits and invests it into a risky asset.

[12] Espinosa et al. (2011) and Kahn and Santos (2005) assume instead that the regulator's political cost exceeds the social cost of a failure. By construction, this assumption leads to consider regulators that are always biased towards forbearance with respect to the first-best policy. Under our assumption, however, regulator's level of forbearance can exceed or fall short with respect to the optimal level conditional on the regulators' incentive structure and the bank's solvency.

At date 1 bank S's liquidity shortfall $v_S$ is publicly observed. The lender of last resort observes in addition privately the solvency signal $u_S$ of bank S and decides about the provision of the emergency liquidity loan. Either bank S receives an emergency loan and continues to operate or bank S is closed. Simultaneously, the non-systemic bank N raises one unit of deposits and invests it into a risky asset.

At date 2 bank N's public liquidity shock $v_N$ is realized. Bank N's solvency signal $u_N$ is privately observed by the lender of last resort. The regulatory agency in charge applies the lender of last resort policy. Bank N is either closed or it remains open if the lender of last resort provides an emergency loan. In case bank S was not liquidated before bank S's risky asset matures simultaneously and its return is realized.

If bank N is still operating at date 3 the return of bank N's risky asset is realized.

# 4 Benchmark case

In our benchmark case we analyze the first- and second-best lending decision within a framework consisting of only one single bank. In this section there is no contagion effect on other financial institutions. As described in section 3 the bank collects one unit of deposits and invests them into a illiquid risky asset with a random return after two periods. After one period of operation the bank faces a random but publicly observable liquidity shock $v$ and can only survive if the lender of last resort provides an emergency loan. The agency in charge of the lender of last resort responsibility uses a perfect but non-verifiable signal about the asset quality to decide whether or not to support the bank. Our benchmark is similar to the model studied in Ponce (2010). The main difference is that we do not consider the deposit insurance corporation in our analysis.

## 4.1 First-best lender of last resort policy

In order to determine the first-best lending decision we assume that the liquidity shock $v$ as well as the solvency signal $u$ are both verifiable.

The expected social welfare from the bank is:

$$W_N = E[\{1_{LLR}(uR-(1-u)c)+(1-1_{LLR})(L-c)] = E[\{1_{LLR}(u(R+c)-L)+(L-c)], \quad (2)$$

where $1_{LLR}$ is equal to 1 if the bank is supported and 0 otherwise. The expected continuation value of the bank including the social cost of a failure after two periods of operation is $(uR-(1-u)c)$. In case the bank is not supported and is liquidated after one period of operation the bank's value net the social cost of the liquidation is $(L-c)$.

Since the bank's liquidation value is constant it is socially optimal to support the bank if the bank's solvency signal is above the threshold $u^*$: $uR-(1-u)c \geq L-c$,

$$u \geq u^* \equiv \frac{L}{R+c}. \tag{3}$$

If the solvency signal falls short of the threshold $u^*$ the bank should not receive emergency liquidity assistance.

## 4.2 Second-best lender of last resort policy

We analyze the second-best lender of last resort policy for the benchmark bank starting with the lending decision of the central banker followed by the provision of liquidity according to the unconditional bailout rule.

### 4.2.1 Central banker as the LLR

Assume that the central banker is the lender of last resort. It will provide the emergency loan to the bank in trouble if the expected utility from supporting the bank exceeds the utility from closing the bank. If the emergency liquidity assistance with an amount of $v$ is provided the emergency loan will be repaid if the supported bank is successful. Otherwise the amount $v$ of the emergency loan is lost. In addition the central banker has to bear the political cost $\alpha c$ of the bank's failure. It follows that the central banker's expected utility from providing the emergency liquidity assistance is equal to $-(1-u)(v+\alpha c)$. If the central banker does not provides the emergency loan the bank is closed and the central banker incurs the political cost $\alpha c$. Consequently, the central banker will support the bank in trouble if the solvency signal is above the threshold $u^{CB}$: $(1-u)(v+\alpha c) \leq \alpha c$,

$$u \geq u^{CB} \equiv \frac{v}{v+\alpha c}. \tag{4}$$

Otherwise the central banker refuses the emergency loan and the bank is liquidated.

### 4.2.2 Unconditional bailout rule

The lending decision given the unconditional bailout rule is applied can be expressed in the following way:

$$u \geq 0 \equiv u^{UBR}. \tag{5}$$

According to the unconditional bailout rule the central banker is instructed to support banks in trouble with an emergency loan independently of the solvency signal $u$.

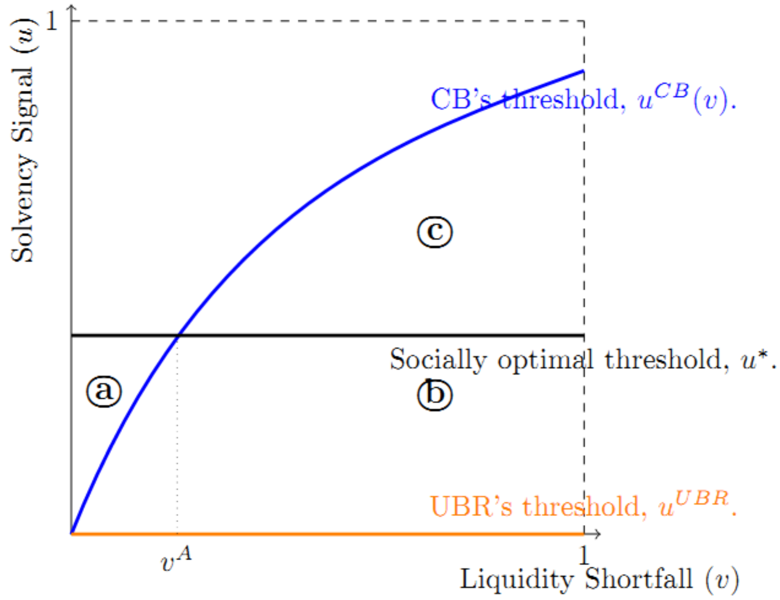


Figure 2: Lending decisions in the benchmark case. It is socially optimal to lend to benchmark banks with solvency signals above $u^{*}$. In region a the central banker (CB) provides socially non-desirable emergency loans; in region c it does not provide socially desirable emergency loans. In regions a and b, socially non-desirable emergency loans are provided by following the unconditional bailout rule (UBR). Let $v^{A} \equiv \dfrac{\alpha cL}{R-L+c}$ be the value for $v$ so that $u^{CB}(v)=u^{*}$. It is immediate that $0 < v^{A} < 1$.

Figure 2 plots the different lending decisions derived above in a $(u,v)$ plane. The first-best emergency liquidity provision requires minimum asset quality $u^{*}$ independent of the size of the liquidity shock. It is therefore a horizontal line. The central banker's threshold of the solvency signal depends on the size of the liquidity shock. With increasing liquidity shortfalls the central banker becomes tougher so that the central banker's lending decision is a concave function passing through the origin. The unconditional bailout rule requires as the first-best liquidity provision a constant level of solvency independent of the size of the liquidity shortfall. But the minimum asset quality requirement is equal to zero. For this reason the unconditional bailout rule lending decision coincide with the abscissa in the $(u,v)$ plane.

The central banker's lending decision is, compared to the first-best provision of liquidity, too soft for small liquidity shortfall and provides socially non-desirable emergency loans. For larger liquidity shocks the central banker is too tough and refuses to provide the socially desirable emergency liquidity assistance. The intuition of this observation is that for very small liquidity shocks close to zero the central banker has an incentive to lend to the bank in trouble. If the central banker does so the expected cost from providing the emergency loan is $(1-u)\alpha c$. If the central banker refuses the emergency loan the bank will be liquidated and the central banker will incur the political cost $\alpha c$ with probability 1 which is larger than $(1-u)\alpha c$. For a larger liquidity shock the exposure of the central banker is more severe so that liquidity is only provided if the solvency signal is sufficiently large.

The unconditional bailout rule is always too soft in comparison with the first-best lending decision because the required asset quality is zero. Only in the origin of the graph the unconditional bailout rule coincide with the central banker's lending decision. For positive liquidity shortfalls the central banker is always tougher and requires a positive solvency signal.

### 4.2.3 Optimal allocation of LLR responsibilities

Following Ponce10 the expected social welfare function (2) given the first-best threshold $u^* \equiv \dfrac{L}{R+c}$ for the provision of an emergency loan can be expressed as:

$$W = E[1_{LLR}(u-u^*)](R+c)+(L-c). \tag{6}$$

To maximize (6) it is sufficient to maximize the normalized expected social welfare:

$$w = E[1_{LLR}(u - u^*)]. \tag{7}$$

From 7 we can derive the normalized expected social welfare given either the central banker acts as the lender of last resort or the unconditional bailout rule is applied:

$$w^{CB}(v) = \int_{u^{CB}(v)}^{1} (u - u^*)\mathrm{d}F(u), \tag{8}$$

$$w^{UBR} = \int_0^1 (u - u^*)\mathrm{d}F(u). \tag{9}$$

Following Ponce (2010) we can shows that these functions have the following properties summarized in Lemma 1.

**Lemma 1** *Assume* $E\left(\tilde{u} \mid u \leq u^{CB}(1)\right) > u^*$. *Then, (1)* $w^{CB}(v)$ *is increasing in* $v$ *if* $v < v^A \equiv \dfrac{\alpha c L}{R - L + c}$, *decreasing if* $v > v^A$, *and has a global maximum at* $v = v^A$; *(2)* $w^{CB}(0) = w^{UBR}$; *and, (3)* $w^{CB}(0) > w^{CB}(1) > 0$.

*Proof.* See Appendix 8.1.

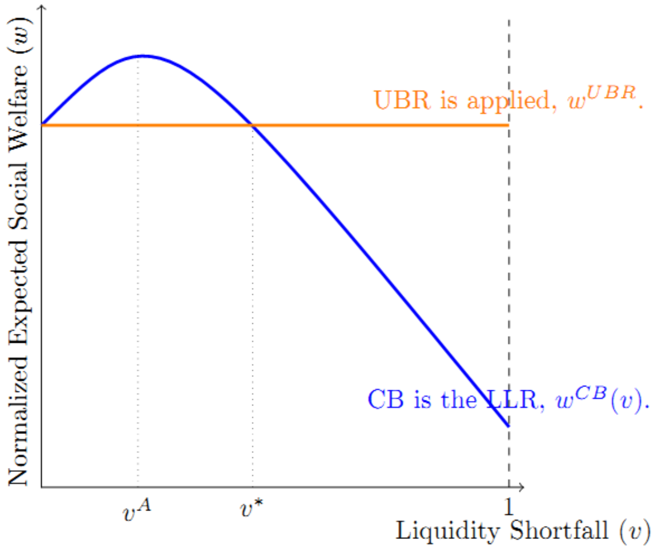


Figure 3: Normalized expected social welfare for the benchmark bank. The optimal

allocation of the lender of last resort activity for the benchmark bank follows the upper envelope of solid functions: for $v < v^*$ the central banker's (CB) decision maximizes $w$; for $v \geq v^*$ the unconditional bailout rule (UBR) maximizes $w$.

Figure 2 visualizes the properties of function (8) and (9) stated in lemma 1. They are presented as a function of the liquidity shortfall. The normalized expected social welfare function given the central banker is the lender of last resort is increasing for $v < v^A$ and decreasing otherwise. At $v^A$ the solvency requirements of the first-best and the central banker coincide so that the emergency liquidity assistance of the central banker corresponds to the first-best provision. For this reason the normalized expected social welfare function has an maximum for $v = v^A$. To the left and the right of $v^A$ the solvency requirement of the central banker differ from the first-best requirement. On the left the central banker is too soft while on the right the central banker is too tough. Therefore, $w^{CB}(v)$ for $v \neq v^A$ is lower than $w^{CB}(v^A)$. The solvency requirement of the unconditional bailout rule has over the whole support of liquidity shocks constant to zero. For this reason the normalized expected social welfare function is a horizontal line.

Since only the liquidity shock $v$ is public information the policy maker will allocate the lender of last resort responsibilities conditional on the size of the liquidity shock to maximize the expected social welfare. As in Ponce10 lemma 1 implies the following second-best optimal allocation:

**Proposition 1** *Assume that* $E\left(\tilde{u} \mid u \leq u^{CB}(1)\right) > u^*$. *It is optimal to allocate the lender of last resort responsibilities to the central banker for liquidity shortfalls below the threshold* $v^* \in (v^A, 1)$. *Otherwise, it is socially optimal to apply the unconditional bailout rule.*

The condition $E\left(\tilde{u} \mid u \leq u^{CB}(1)\right) > u^*$ implies that the asset quality of a random bank is more likely to be of average quality (i.e. $u \in [u^*, u^{CB}(1)]$) than of low quality (i.e. $u \in [0, u^*]$). In the interval $[0, u^{CB}(1)]$ the central banker might not provide socially desirable emergency loan depending on the size of the liquidity shortfall. But the average bank has a sufficient quality according to the first-best lending decision. For this reason, it is welfare-enhancing to apply the unconditional bailout rule for large liquidity shocks because for these shocks it is more likely that the central banker will be too restrictive and not provide socially desirable emergency loans. For small liquidity shocks the central banker's lending decision is the one closest to the first-best solution so that the allocation of lender of last resort responsibilities to the central banker for small liquidity shocks is welfare enhancing.

# 5 Financial system with a systemic bank

In this section we study the optimal lender of last resort policy for a financial system with a systemic and a non-systemic bank as described in section 3. In order to determine the optimal allocation of responsibilities for the systemic bank we solve the model backwards starting with the non-systemic bank followed by the systemic bank.

We define the following indicator variables with a value equal to one in case the below-mentioned conditions hold:

- $1_S^{SS} = 1$ if systemic bank S succeeds at date 2.
- $1_S^{SF} = 1$ if systemic bank S fails at date 2 or was closed at date 1.
- $1_S = 1$ if LLR loan is provides to systemic bank S.
- $1_N^{SS} = 1$ if LLR loan is provided to non-systemic bank N given systemic bank S succeeded.
- $1_N^{SF} = 1$ if LLR loan is provided to non-systemic bank N given systemic bank S failed at date 2 or was closed at date 1.

## 5.1 Lender of last resort policy for the non-systemic bank

### 5.1.1 First-best

For the determination of the socially optimal allocation of the LLR responsibilities we assume that the liquidity shock $v_S$ and the solvency signal $u_S$ are both public information and verifiable. The expected social welfare from bank N is:

$$W_N = E\{1_S^{SS}[\{1_N^{SS}(u_N R-(1-u_N)c)+(1-1_N^{SS})(L-c)] \\ +1_S^{SF}[1_N^{SF}(u_N(R-\gamma)-(1-u_N)c)+(1-1_N^{SF})(L-c)]\}, \tag{10}$$

where the first term of this expression is the expected social welfare given a successful systemic bank (case SS). If the non-systemic bank is supported with an emergency loan the bank succeeds with probability $u_N$ and yields a return $R$. A failure occurs with a probability $(1-u_N)$ and causes social cost $c$. If bank N is not supported the bank will be liquidated and the liquidation value $L$ will be realized. The closure causes social cost of $c$.

The second term of (10) is the expected social welfare in case bank S was liquidated at date 1 or its risky asset failed at date 2 (case SF). If bank N receives an emergency loan its risky asset succeeds with probability $u_N$ but yield only a return $R-\gamma$. The asset fails with probability $(1-u_N)$ which causes social costs of $c$. If the emergency loan is refused bank N is liquidated and a liquidation value of $L$ is realized. A liquidation causes social cost of $c$.

For the determination of the first-best lending decision the thresholds on the solvency signal $u_N$ are derived separately for both states of the systemic bank S (case SS and case SF). First, the case of a successful bank S is analyzed. It is optimal to provide an emergency loan to bank N if the expected social welfare from bank N's continuation exceeds the social welfare of bank N's liquidation. The social optimal lending decision to bank N in case SS is:

$$u_N R-(1-u_N)c \geq L-c,$$

$$u_N \geq u_N^{SS} \equiv \frac{L}{R+c}, \tag{11}$$

which is equivalent with the first-best lending decision in our benchmark case. If the solvency signal $u_N$ is below $u_N^{SS}$ it is not socially optimal to provide the emergency liquidity assistance.

If the systemic bank S fails it is optimal to provide emergency liquidity assistance to bank N if:

$$u_N(R-\gamma)-(1-u_N)c \geq L-c,$$

$$u_N \geq u_N^{SF} \equiv \frac{L}{R+c-\gamma}. \tag{12}$$

In equation (12) we observe the negative impact of the systemic bank's failure on the non-systemic bank's asset return in threshold $u_N^{SF}$. Due to the lower asset return the first-best lending decision in case SF is tougher compared to the threshold for case SS in equation (11).

### 5.1.2 Central banker as the LLR

The central banker will only support the non-systemic bank N if the expected cost from providing the emergency loan is lower than the cost of closing bank N immediately. The central banker's expected cost of an emergency loan to bank N has two components: First, the expected losses of the liquidity injection $v_N$ and second the political cost $\alpha c$ due to a failure of bank N. If the central banker does not provide the liquidity bank N will be closed and the central banker will incur the political cost $\alpha c$ for the bank failure. The state of bank S has no impact on the central banker's expected cost, because $R-\gamma>1$. Even if the systemic bank fails the successful non-systemic bank will be able to repay the emergency loan. The central banker's expected utility from the lender of last resort activities is:

$$\begin{aligned}B_N &= 1_S^{SS}[\{1_N^{SS}(-(1-u_N)(\alpha c+v_N))+(1-1_N^{SS})(-\alpha c)] \\ &+1_S^{SF}[1_N^{SF}(-(1-u_N)(\alpha c+v_N))+(1-1_N^{SF})(-\alpha c)].\end{aligned} \tag{13}$$

The central banker will provide the emergency liquidity if:

$$u_N(\alpha c+v_N) \geq v_N,$$

$$u_N \geq u_N^{CB}(v_N) \equiv \frac{v_N}{v_N+\alpha c}, \tag{14}$$

which is equivalent to the central banker's lending decision in the benchmark case. If the solvency signal $u_N$ is below $u_N^{CB}$ the central banker will refuse the emergency loan and the non-systemic bank will be closed.

### 5.1.3 The unconditional bailout rule

The lending decision given the unconditional bailout rule is applied can be expressed in the following way:

$$u_N \geq 0 \equiv u_N^{UBR}. \tag{15}$$

It implies that banks with a positive liquidity shock $v_N$ will always be supported independent of the solvency signal $u_N$.

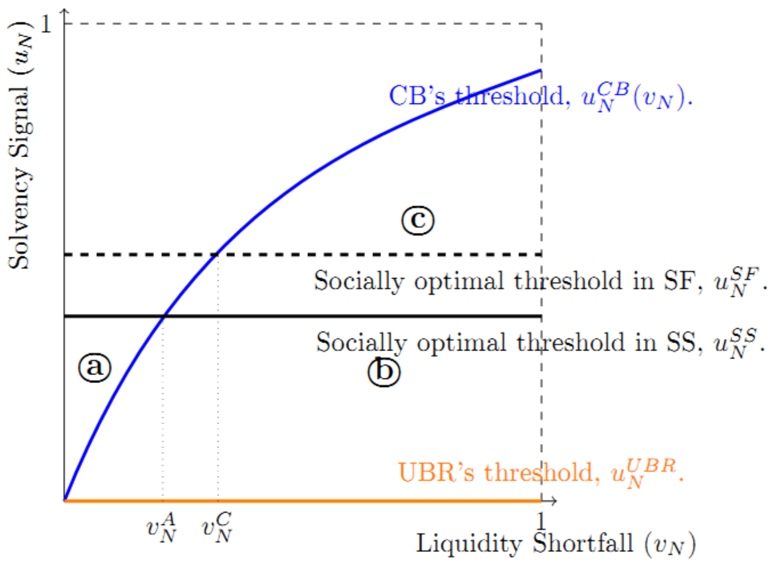


Figure 4: Lending decisions for the non-systemic bank. It is socially optimal to lend to non-systemic banks with solvency signals above $u_N^i$ for $i \in \{SS, SF\}$. In region a the central banker (CB) provides socially non-desirable emergency loans; in region c she does not provide socially desirable emergency loans. In regions a and b, socially non-desirable emergency loans are provided by following the unconditional bailout rule (UBR). In state $SS$ the systemic bank's asset was successful while in state $SF$ the systemic bank either was liquidated or its asset failed. Let $v_N^A \equiv \frac{\alpha cL}{R-L+c}$ be the value for $v_N$ so that $u_N^{CB}(v_N) = u_N^{SS}$ and $v_N^C \equiv \frac{\alpha cL}{R-L+c-\gamma}$ be the value for $v_N$ so that $u_N^{CB}(v_N) = u_N^{SF}$ İ t is immediate that $0 < v_N^A < v_N^C < 1$.

Figure 4 shows the liquidity provision thresholds for the non-systemic bank N defined above. The first-best lending decision depends on the state of bank S but is independent of the size of the liquidity shortfall $v_N$. If bank S fails or was liquidated the first-best lending decision is more restrictive and requires a higher solvency signal $u_N^{SF}$. For this reason the first-best lending decision in case SF is above the threshold in case SS. The central banker's lending decision is independent of bank S's state. It only coincides with the socially optimal lending decision for a liquidity shock of size $v_N^A$ ($v_N^C$) in case SS (SF). Since the central banker's expected utility is decreasing with the size of the required emergency loan the central banker's lending decision becomes more restrictive with increasing liquidity shocks. The unconditional bailout rule always provides the emergency loan so that the lending decision in the (v,u) plane coincide with the abscissa.

The comparison of the policies for the non-systemic bank N with the benchmark case yields the following proposition:

**Proposition 2** *The first-best lender of last resort policy for the non-systemic bank is more restrictive than in the benchmark case if the systemic bank was liquidated or failed i.e. some banks that where supported by emergency liquidity assistance in the benchmark case are not supported now. Otherwise the first-best lending decision for non-systemic bank is identical to the benchmark case. The lending decisions of the central banker and the unconditional bailout rule to the non-systemic bank are identical to with the benchmark case.*

*Proof.* See Appendix 8.2.

#### 5.1.4 Optimal allocation

Since the first-best lending decision for the non-systemic bank differs between between cases $SS$ and $SF$ we will study the optimal allocation of LLR responsibilities for both cases separately. On the basis of case $SS$ we illustrate our approach to define the optimal second-best allocation of LLR responsibilities. As in the benchmark case the expected social welfare in (10) given the socially optimal threshold to provide emergency liquidity $u_N^{SS} \equiv \dfrac{L}{R+c}$ can be expressed as:

$$W_N^{SS} = E[1_N^{SS}(u_N - u_N^{SS})](R+c) + (L-c). \tag{16}$$

It is sufficient to maximize the normalized social welfare:

$$w_N^{SS} = E[1_N^{SS}(u_N - u_N^{SS})] \tag{17}$$

in order to obtain the maximum of the social welfare in equation (16).

As the approach for $\nu = SF$ is analogous it follows for $\nu \in \{SF, SS\}$ that the normalized expected social welfare functions given the central banker is the lender of last resort or the unconditional bailout rule is applied are:

$$w_N^{CB,\nu}(v_N) = \int_{u_N^{CB}(v_N)}^{1} (u_N - u_N^{\nu}) \mathrm{d}F(u), \tag{18}$$

$$w_N^{UBR,\nu} = \int_0^1 (u_N - u_N^{\nu}) \mathrm{d}F(u). \tag{19}$$

Lemma 2 follows Ponce (2010) results adapted to the model studied in this paper and proves some properties of the normalized expected social welfare functions (18) and (19).

**Lemma 2** *Assume that* $E\left(\tilde{u}_N \,|\, u_N \leq u_N^{CB}(1)\right) > u_N^{SF}$. *Then, (1) if the systemic bank succeeded,* $\nu = SS$ *(respectively failed,* $\nu = SF$*), then (i)* $w_N^{CB,SS}(v_N)$ *(*$w_N^{CB,SF}(v_N)$*) is increasing in* $v_N$ *if* $v_N < v_N^A \equiv \dfrac{\alpha c L}{R - L + c}$ *(respectively* $v_N < v_N^C \equiv \dfrac{\alpha c L}{R - L + c - \gamma}$*), (ii) decreasing if* $v_N > v_N^A$ *(respectively* $v_N > v_N^C$*), and (iii) has a global maximum at* $v_N = v_N^A$ *(respectively at* $v_N = v_N^C$*); (2)* $w_N^{CB,\nu}(0) = w_N^{UBR,\nu}$*; (3)* $w_N^{CB,\nu}(0) > w_N^{CB,\nu}(1) > 0 \;\forall\; \nu \in \{SS, SF\}$.

*Proof.* See Appendix 8.3.

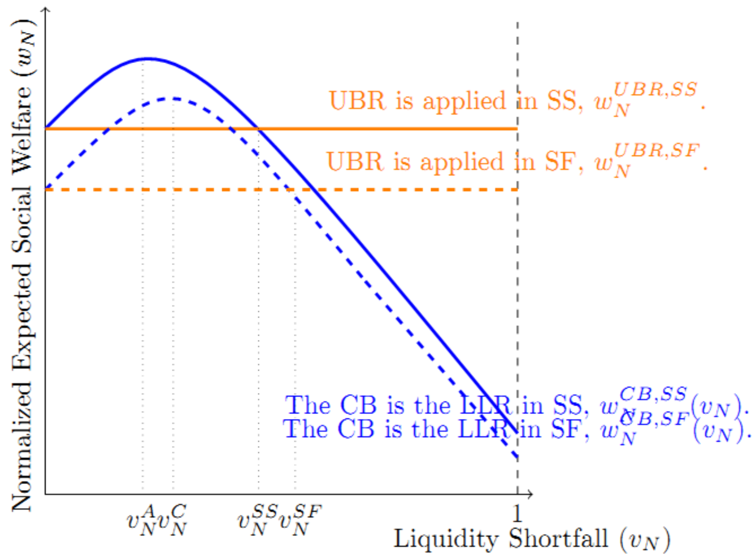


Figure 5: Normalized expected social welfare for the non-systemic bank. The optimal allocation of the lender of last resort activity for the non-systemic bank follows the upper envelope of solid functions in case the systemic bank survives and is successful. Otherwise it follows the upper envelope of the dashed functions: for $v_N < v_N^i$ the central banker's (CB) decision maximizes $w_N$; for $v_N \geq v_N^i$ the unconditional bailout rule (UBR) maximizes $w_N$ for $i \in \{SS, SF\}$. In state $SS$ the systemic bank's asset was successful while in state $SF$ the systemic bank either was liquidated or its asset failed.

Figure 4 presents the normalized expected social welfare functions (18) and (19) and explains the properties proven in the Lemma 2. The normalized expected social welfare given the central banker is the lender of last resort increases for liquidity shortfall smaller than $v_N^A$ ($v_N^C$) in case SS (SF) because the central banker's lending decision converges to the first-best provision of liquidity. For liquidity shocks above these thresholds the normalized expected social welfare decreases because the central banker becomes more restrictive and diverges from the first-best provision of liquidity. The normalized expected social welfare function given the unconditional bailout rule is applied is horizontal because the unconditional bailout rule provides an emergency loan independent of the size of the liquidity shock. Due to the concave function the normalized expected social welfare function given the central banker is the lender of last resort, intersects with the normalized expected social welfare function if the unconditional bailout rule is applied in case SS (SF) for two liquidity shocks: 0 and $v_N^{SS}$ (0 and $v_N^{SF}$).

Since only the liquidity shock $v_N$ is verifiable the policy maker will allocate the lender of last resort responsibilities according to the size of the liquidity shortfall in order to maximize the expected social welfare. Lemma 2 implies the following second-best optimal allocation:

**Proposition 3** *Assume that* $E\left(\tilde{u}_N \mid u_N \leq u_N^{CB}(1)\right) > u_N^{SF}$. *If the systemic bank succeeded,* $v = SS$ *(respectively failed,* $v = SF$*), there exists a threshold for the liquidity shortfall of the non-systemic bank* $v_N^{SS} \in (v_N^A, 1)$ *(respectively* $v_N^{SF} \in (v_N^C, 1)$*) so that it is optimal to allocate the lender of last resort responsibilities for the non-systemic bank to the central banker for liquidity shortfalls below the threshold and to apply the unconditional bailout rule for liquidity shortfalls above it.*

Proposition 3 can be explained as followed. For large liquidity shock the central banker's lending decision is too restrictive. Given condition $E\left(\tilde{u}_N \mid u_N \leq u_N^{CB,v}(1)\right) > u_N^{SF}$ it is more likely that the asset of a random non-systemic bank is of average quality (i.e. $u \in [u_N^{SF}, u_N^{CB}(1)]$) than of low quality (i.e. $u \in [0, u_N^{SF}]$). With increasing liquidity shocks it is more likely that a non-systemic bank for which liquidity support is social optimal does not receive an emergency loan from the central banker compared to an unconditional bailout of a non-systemic bank with low quality assets. Therefore, the policy maker chooses to apply the unconditional bail out rule for large liquidity shortfalls. For small liquidity shock the central banker's threshold is closer to the socially optimal one than the unconditional bailout rule. Therefore, the LLR responsibilities for small liquidity shock are allocated to the central banker.

We can show that the existence of the systemic bank provides a rationale for the central banker to act as a lender of last resort with an extended mandate. Proposition 4 summarizes this finding:

**Proposition 4** *The central banker should act as a lender of last resort in a larger* *range of liquidity shortfalls of the non-systemic bank when the systemic bank failed than when it succeeded (i.e. $v_N^{SS} < v_N^{SF}$).*

*Proof.* See Appendix 8.4.

This result can be explained in the following way. With a failure of the systemic bank the expected return of the non-systemic bank falls so that for all liquidity shocks the socially optimal threshold for the provision of the emergency loan increases. Since the central banker become less forbearing with increasing liquidity shocks the socially optimal lending decision is closer to the central banker's one for a larger interval of liquidity shocks. However, for very large liquidity shocks the central banker is still too severe, so that the unconditional bailout rule still maximizes the expected social welfare in this interval.

## 5.2 Lender of last resort policy for the systemic bank

### 5.2.1 First-best

As for the non-systemic bank we determine the first-best provision of emergency liquidity assistance by the comparison of the expected social welfare from supporting and not supporting the bank given that the liquidity shock $v_S$ as well as the solvency shock $u_S$ are both verifiable. The expected social welfare is:

$$W_S = E\{1_S[u_S R-(1-u_S)c+W_N^{SC}]+(1-1_S)[L-c+W_N^{SL}]\},$$

$$W_S = E\{1_S[u_S(R+c)-L+W_N^{SC}-W_N^{SL}]+L-c+W_N^{SL}\}, \tag{20}$$

where the first term is the social welfare given the systemic bank receives the emergency liquidity assistance. In this case the social welfare consists of the systemic bank's expected continuation value at date 2 net of social cost ($u_S R-(1-u_S)c$) and the expected social welfare of the non-systemic bank $W_N^{SC}$ given the systemic bank continues to operate. The latter has to be considered because as we showed in section 5.1 the state of the systemic bank has an impact on the emergency liquidity provision for the non-systemic bank. The expected social welfare from the non-systemic bank given the systemic bank continues to operate is:

$$W_N^{SC} = u_S E\{1_N^{SS}(u_N R-(1-u_N)c)+(1-1_N^{SS})(L-c)\}$$ (21)
$$+(1-u_S)E\{1_N^{SF}(u_N(R-\gamma)-(1-u_N)c)+(1-1_N^{SF})(L-c)\},$$

which consists of the expected social welfare from the non-systemic bank given the systemic bank is successful or fails at date 2. If the systemic bank is successful the non-systemic bank's expected continuation value net of the social cost of failure is $u_N R-(1-u_N)c$. If the emergency loan is refused the non-systemic bank will be liquidated so that the social welfare is equal to the liquidation value net of the social cost due to bank failure $L-c$. In case the systemic bank fails at date 2 the non-systemic bank's expected continuation value net of the expected social cost of a bank failure is $u_N(R-\gamma)-(1-u_N)c$. The liquidation value of the non-systemic bank net of the social cost of a bank failure is $L-c$.

The second term of equation (20) is the social welfare in case the systemic bank is closed. The liquidation value net of social cost of a failure at date 1 is $L-c$. As before the expected social welfare from the non-systemic bank given the systemic bank was closed has to be considered. This is:

$$W_N^{SL} = E\{1_N^{SF}(u_N(R-\gamma)-(1-u_N)c)+(1-1_N^{SF})(L-c)\},$$ (22)

where the first term corresponds to the situation when the non-systemic bank receives emergency liquidity assistance while the second term refers to the situation when the emergency loan is refused. If the non-systemic bank is supported its expected continuation value net of the social cost of a bank failure is $u_N(R-\gamma)-(1-u_N)c$. If the non-systemic bank is not supported the liquidation value net of the social cost of a bank failure at date 2 is $L-c$.

We define:

$$W_N^{\Delta} = E\{(1_N^{SS}-1_N^{SF})(u_N(R+c)-L)+1_N^{SF}u_N\gamma\} \geq 0,$$ (23)

so that

$$W_N^{SC}-W_N^{SL} = u_S W_N^{\Delta}.$$ (24)

Given (20) and (24) it is social optimal to provide the emergency loan to the systemic bank if:

$$u_S(R+c+W_N^{\Delta}) \geq L,$$

$$u_S \geq u_S^* \equiv \frac{L}{R+c+W_N^{\Delta}}. \tag{25}$$

Is the solvency signal below $u_S^*$ the systemic bank should be closed because it is socially not optimal to provide the emergency loan.

### 5.2.2 Central banker as the LLR

Suppose the central banker is the lender of last resort for the systemic bank. If the central banker engages in the emergency liquidity assistance but the systemic bank fails the central banker loses the liquidity injection $v_S$ and incurs the political costs $\alpha c$. In addition the utility from the non-systemic bank N given the systemic bank continues to operate $B_N^{SC}$ has to be considered because the state of the systemic bank influences the expected profitability of the bank N and the central banker's responsibilities as a lender of last resort for the non-systemic bank. If the central banker refuses to support the systemic bank the central banker will not provide any liquidity. Bank S will be closed. In this situation the central banker's cost consists only of the political cost $\alpha c$ and the central banker's utility from the non-systemic bank N given the closure of the systemic bank $B_N^{SL}$. Thus, the central banker's expected utility from its lender of last resort responsibilities for the systemic bank S is:

$$B_S = E\{1_S[-(1-u_S)(\alpha c+v_S)+B_N^{SC}]+(1-1_S)[-\alpha c+B_N^{SL}]\},$$

$$B_S = E\{1_S[u_S(v_S+\alpha c)-v_S+B_N^{SC}-B_N^{SL}]-\alpha c+B_N^{SL}\}. \tag{26}$$

The central banker's utility from the non-systemic bank N given the systemic bank continues to operate is:

$$\begin{aligned} B_N^{SC} = {} & u_S\int_0^{v_N^{SS}}\left[\int_0^{u_N^{CB}(v_N)}-(\alpha c)dF(u)+\int_{u_N^{CB}(v_N)}^{1}-(1-u_N)(\alpha c+v_N)dF(u)\right]dG(v_N) \\ & +(1-u_S)\int_0^{v_N^{SF}}\left[\int_0^{u_N^{CB}(v_N)}-(\alpha c)dF(u)+\int_{u_N^{CB}(v_N)}^{1}-(1-u_N)(\alpha c+v_N)dF(u)\right]dG(v_N). \end{aligned} \tag{27}$$

The first term reflects the situation when the systemic bank is successful. The second term refers to the situation when the systemic bank fails. $v_N^{SS}$ and $v_N^{SF}$ are the optimal second-best liquidity shocks below which the central banker is the lender of last resort for the non-systemic bank as defined in Proposition 3. When the central banker is responsible for the provision of the emergency loan the central banker will only support the bank in trouble if the solvency signal is above the threshold $u_N^{CB}$. In case the non-systemic bank fails while being supported the central banker will lose the emergency loan $v_N$ and incur the political cost of bank failure $\alpha c$. Below the solvency threshold the central banker will never support bank N and incur the political cost $\alpha c$.

The utility from the non-systemic bank N given a closure of the systemic bank S is:

$$B_N^{SL} = \int_0^{v_N^{SF}} \left[ \int_0^{u_N^{CB}(v_N)} -(\alpha c) dF(u) + \int_{u_N^{CB}(v_N)}^{1} -(1-u_N)(\alpha c + v_N) dF(u) \right] dG(v_N), \quad (28)$$

where $v_N^{SF}$ is the non-systemic bank's liquidity shock below which the central banker is the lender of last resort. The central banker will refuse the emergency loan if the non-systemic bank's solvency signal is below $u_N^{CB}(v_N)$. In this situation the non-systemic bank will be closed and the central banker will incur the political cost $\alpha c$. The central banker supports bank N given the solvency signal is above the threshold $u_N^{CB}(v_N)$. If the non-systemic bank fails the emergency loan $v_N$ will not be repaid and the central banker will incur additionally the political cost $\alpha c$.

We define:

$$B_N^{\Delta} = \int_{v_N^{SS}}^{v_N^{SF}} \left[ \int_0^{u_N^{CB}(v_N)} (\alpha c) dF(u) + \int_{u_N^{CB}(v_N)}^{1} (1-u_N)(\alpha c + v_N) dF(u) \right] dG(v_N) \geq 0,$$

so that

$$B_N^{SC} - B_N^{SL} = u_S B_N^{\Delta}. \quad (29)$$

Given (26) and (29) the central banker lends to bank S if:

$$u_S (v_S + \alpha c + B_N^{\Delta}) \geq v_S,$$

$$u_S \geq u_S^{CB}(v_S) \equiv \frac{v_S}{v_S + \alpha c + B_N^{\Delta}}. \quad (30)$$

The central banker will refuse the emergency loan if the solvency signal is below $u_S^{CB}(v_S)$.

The solvency threshold of the central banker is lower compared to the threshold in the benchmark case in equation (4). The central banker incorporates into the lending decision for the systemic bank the effect of its behaviour in respect of its responsibilities towards the non-systemic bank. Since the central banker has more responsibilities for the non-systemic bank if the systemic bank fails it will be less strict with the latter one in order to avoid the extended mandate.

### 5.2.3 Unconditional bailout rule

The lending decision given the unconditional bailout rule is applied can be expressed in the following way:

$$u_S \geq 0 \equiv u_S^{UBR}. \tag{31}$$

Banks with a positive liquidity shock $v_S$ will always be supported regardless of the solvency signal $u_S$.

Proposition 5 summaries the effect of the systemic bank on the lending decision compared to the benchmark case.

**Proposition 5** *The first-best lender of last resort policy for the systemic bank is softer compared to the benchmark case, i.e. some banks that do not receive support in the benchmark should receive support if they are systemic. The central banker's lending decision for the systemic bank is also less strict compared to the benchmark case while the unconditional bailout rule remains unchanged.*

*Proof.* See Appendix 8.5.

The intuition of the softer first-best lender of last resort policy for systemic banks can be explained by the negative impact of a systemic bank's failure on the expected return of the non-systemic bank. The first-best lender of last resort policy is not only driven by the comparison between the expected continuation value and the liquidation value of the systemic bank but also considers the continuation value of the non-systemic bank in both states of the systemic bank. If the systemic bank is not supported the expected profitability of the non-systemic bank is reduced and the social welfare is harmed. For this reason, the first-best lender of last resort policy is more forbearing for the systemic bank compared to the benchmark case.

The central banker is also more lenient compared to the benchmark case. This is due to the second-best optimal allocation of the responsibilities for the non-systemic bank. A closure of the systemic bank implies more responsibilities for the central banker as a lender of last resort for the non-systemic bank. The central banker is hence exposed to a larger expected loss from the lender of last resort activities because his mandate is extended. As a consequence the expected utility decreases. In order to avoid this negative impact on its utility the central banker is biased towards forbearance for the systemic bank.

#### 5.2.4 Optimal allocation

As above the expected social welfare function for the systemic bank S in equation (??) given the first-best liquidity provision threshold $u_S^* = \frac{L}{R+c+W_N^{\Delta}}$ can be expressed as:

$$W_S = E[1_S(u_N - u_S^*)](R+c) + (L - c + W_N^{SL}). \tag{32}$$

It is sufficient to maximize the normalized expected social welfare:

$$w_S = E[1_S(u_N - u_S^*)] \tag{33}$$

in order to maximize equation (32). The normalized expected social welfare functions given the central banker is the lender of last resort or the unconditional bailout rule is applied are stated below:

$$w_S^{CB}(v_N) = \int_{u_S^{CB}(v_S)}^{1} (u_S - u_S^*)\mathrm{d}F(u), \tag{34}$$

$$w_S^{UBR} = \int_0^1 (u_S - u_S^*)\mathrm{d}F(u). \tag{35}$$

We can show that these functions have the following properties summarized in Lemma 3.

**Lemma 3** *Assume that* $E\left(\tilde{u}_S \mid u_S \le u_S^{CB}(1)\right) > u_S^*$. *Then, (1)(i)* $w_S^{CB}(v_S)$ *is increasing* *in* $v_S$ *if* $v_S < v_S^A \equiv \dfrac{L(\alpha c + B_N^\Delta)}{R - L + c + W_N^\Delta}$*, (ii) decreasing if* $v_S > v_S^A$*, and (iii) has a global maximum at* $v_S = v_S^A$*; (2)* $w_S^{CB}(0) = w_S^{UBR}$*; (3)* $w_S^{CB}(0) > w_S^{CB}(1) > 0$.

*Proof.* See Appendix 8.6.

The shape of the normalized expected social welfare functions are as in the benchmark and as for the non-systemic bank. The difference here is that for the systemic bank the global maximum at $v_S = v_S^A$ is also determined by the non-systemic bank.

The policy maker will allocate the lender of last resort responsibilities among the central banker and the unconditional bailout rule conditional on the liquidity shortfall in order to maximize the normalized expected social welfare because only the liquidity shortfall is publicly available and verifiable. Lemma 3 implies the following proposition:

**Proposition 6** *Assume* $E\left(\tilde{u}_S \mid u_S \le u_S^{CB}(1)\right) > u_S^*$*. There exist an liquidity shortfall* $v_S^* \in \{v_S^A, 1\}$ *so that it is optimal to allocate the lender of last resort responsibilities to the central banker for all liquidity shock smaller than* $v_S^*$*. Above* $v_S^*$ *the it is optimal to apply the unconditional bailout rule.*

The intuition of proposition 6 is as follows. Condition $E\left(\tilde{u}_S \mid u_S \le u_S^{CB}(1)\right) > u_S^*$ implies that the asset of a random systemic bank is more likely to be of an average quality (i.e. $u \in [u_S^*, u_S^{CB}(1)]$) than of low quality (i.e. $u \in [0, u_S^*]$). It is more likely that the central banker does not provide the socially optimal emergency loan to a systemic bank if the liquidity shortfall is larger because the central banker's lending decision is too restrictive. For this reason the policy maker chooses to apply the unconditional bail out rule for large liquidity shocks. As above it is not always optimal to support illiquid banks unconditionally. For small liquidity shock the central banker is still more restrictive than the unconditional bailout rule but the central banker's threshold is closer to the socially optimal one. Therefore, the LLR responsibilities for small liquidity shock are allocated to the central banker.

If the condition $E\left(\overset{\sim}{u_S} \mid u_S \le u_S^{CB}(1)\right) > u_S^*$ is not satisfied the systemic bank's solvency is on average insufficient to receive an emergency loan from a first-best point of view. Instead of being supported the systemic bank should be closed and liquidated. In this situation the policy maker prefers not to apply the unconditional bailout rule because too many low quality systemic banks would receive socially non-optimal emergency liquidity loans. The central banker will be responsible for the entire set of liquidity shocks because welfare losses from closing systemic banks with a sufficient first-best solvency are overcompensated by the restrictive lending decision of the central banker for small liquidity shocks.

Proposition 6 defines the threshold on the liquidity shock for the systemic bank where the responsibility is transfered from the central banker to the unconditional bailout rule. For the determination of the range of action for the central banker and the unconditional bailout there exist two counteracting effects. First, the central banker's lending decision is less strict for the systemic bank compared to the benchmark case because the central banker incorporates the consequences of the systemic bank's collapse into its responsibilities for the non-systemic bank. Keeping the first-best lending decision constant this leads to more responsibilities for the central banker because for a larger interval of liquidity shocks the central banker's behavior is closer to the first-best provision of liquidity. Second, the first-best lending decision itself is more forbearing with the systemic bank because the negative effect of the systemic bank's collapse on the non-systemic bank's profitability is taken into consideration. Other things being equal the lower solvency requirement of the first-best solution leads to less responsibilities for the central banker because the interval of liquidity shortfalls where the central banker is too stringent increases. The parameter constellation of the model defines which of the two effects prevails. But the implications of these parameters for the effect of the systemic risk on both lending decisions are ambiguous so that the overall effect of the systemic risk on the optimal second-best allocation for the systemic bank is undetermined.

# 6 Extension

Until now the policy maker could allocate the lender of last resort responsibilities only between the central banker and the unconditional bailout rule. In his section we introduce the deposit insurer into the model so that the set of available policy instruments for the policy maker is enlarged. We do this to verify that the optimal allocation of responsibilities for the systemic and non-systemic bank derived above was not determined by the truncated set of policy instruments considered above.

The deposit insurer has to carry out the deposit insurance function. This obliges it to compensate depositors if a bank fails. It has two options to raise funds for the compensation payments. First, it has access to the failed bank's asset and can realize the liquidation value $L$. Second, it is funded by banks through deposit insurance premiums. For simplicity, we assume that the deposit insurance premium is normalized to zero.

When appointed as the lender of last resort the deposit insurer observe the solvency signal $u$. In case the bank in trouble fails or is liquidated during its mandate it incurs political cost $\beta c$. As for the central banker we assume that the deposit insurer cares about the expected value of its final wealth net of political cost incurred in case of a bank failure.

In the following we will analyze the lending decision of the deposit insurer in the benchmark case as well as for the non-systemic and systemic bank and present the effect for the optimal second-best allocation.

## 6.1 Lender of last resort policy in the benchmark case

Suppose that the deposit insurer has to decide in the benchmark case about the provision of an emergency loan to a bank hit by a liquidity shock $v$. The deposit insurer will support the bank if the expected utility from supporting exceeds the utility from liquidating the bank. The deposit insurer's utility from lending to the bank the amount $v$ depends on whether the supported bank is successful or not. If the bank is successful the emergency loan $v$ is repaid. When the bank fails the deposit insurer loses the emergency loan $v$. In addition it has to compensate the remaining depositors $(1-v)$ and incurs the political cost of the bank's failure $\beta c$. The expected utility from supporting the bank is therefore equal to $-(1-u)(1+\beta c)$. If the deposit insurer does not support the bank in trouble it will incur the political cost of the bank's failure $\beta c$ and has to compensate all depositors but can realize the liquidation value $L$. So the utility of the deposit insurer will be $L-1-\beta c$. The deposit insurer will lend the amount $v$ to the bank if:

$$-(1-u)(1+\beta c) \leq L-1-\beta c,$$

$$u \geq u^{DI} \equiv \frac{L}{1+\beta c}. \tag{36}$$

The deposit insurer's lending decision does not depend on the size of the liquidity shortfall because the liability of the deposit insurer is bounded above by the amount of deposits. The exposure is only reduced by the liquidation value in case of a closure. It is not affected by the substitution of deposits by an emergency loan. Comparing the first-best lending decision in the benchmark case in equation (3) with (36) it is obvious that the deposit insurer requires a higher solvency signal than the first-best lending decision in the benchmark case. Hence the deposit insurer is more restrictive and does not provide socially optimal emergency loans.

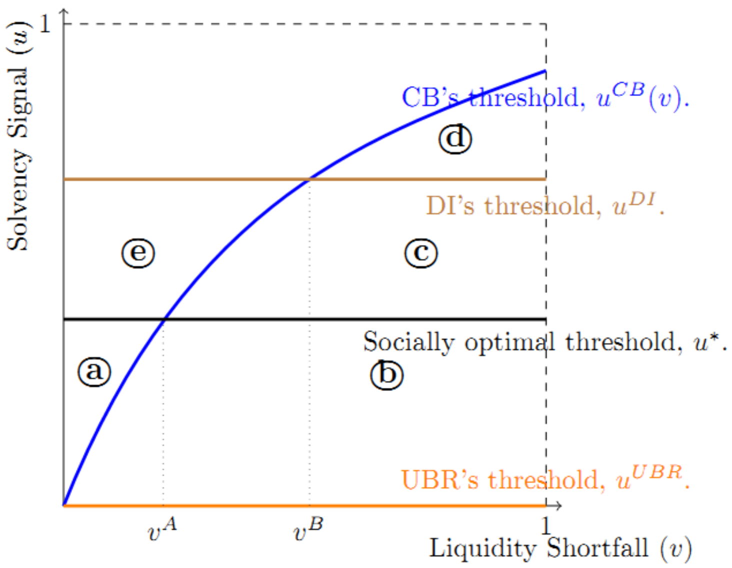


Figure 6: Lending decisions in the benchmark case. It is socially optimal to lend to benchmark banks with solvency signals above $u^*$. In region a the central banker (CB) provides socially non-desirable emergency loans; in regions c and d it does not provide socially desirable emergency loans. In regions c and e the deposit insurer (DI) does not provide socially desirable emergency loans. In regions a and b, socially non-desirable

emergency loans are provided by following the unconditional bailout rule (UBR). Let $v^A \equiv \frac{\alpha cL}{R-L+c}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B \equiv \frac{\alpha cL}{1-L+\beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is obvious that $0 < v^A < v^B$. Moreover, $c < \frac{1-L}{L}$ implies that $v^B < 1$.

Figure 6 presents the lending decision of the deposit insurer in comparison with that of the first-best solution, the central banker and the unconditional bailout rule. On the horizontal axis we find the liquidity shock while the solvency signal is plotted on the ordinate. The agency's lending decisions and the unconditional bailout rule do not coincide with each other. Both are constant over the whole range of liquidity shocks. They also do not match with the first-best solution. The deposit insurer is always too stringent compared to the first-best liquidity provision while the unconditional bailout rule is always too lenient. As mentioned above the central banker is too soft for small liquidity shocks and too tough for larger liquidity shortfalls compared to the first-best provision. Ponce (2010) points out that the main reasons for the divergence between the agencies' lending decisions is due to the differing impacts of an emergency loan on the agencies' utility. While the deposit insurer has to compensate all depositors of a collapsed bank, but can realize the liquidation value the central banker's exposure is restricted to the amount of the emergency loan. Additionally, the different weights of the political cost of a failure ($\alpha$, $\beta$) drive the lending decisions apart.

In order to determine the second-best optimal allocation when the deposit insurer is considered we derive the normalized expected social welfare function given the deposit insurer acts as the lender of last resort from the expected social welfare function in (2) and the first-best threshold $u^* \equiv \frac{L}{R+c}$:

$$w^{DI} = \int_{u^{DI}}^{1} (u - u^*) \mathrm{d}F(u). \tag{37}$$

Lemma 4 proves some properties of (??) and relates it to the normalized expected social welfare functions of the central banker and the unconditional bailout rule in equation (??) and (??):

**Lemma 4** *Assume* $E\left(\tilde{u} \mid u \le u^{DI}\right) > u^*$. *Then, (1)* $w^{CB}(v)$ *is increasing in* $v$ *if* $v < v^A \equiv \dfrac{\alpha cL}{R - L + c}$, *decreasing if* $v > v^A$, *and has a global maximum at* $v = v^A$; *(2)* $w^{CB}(0) = w^{UBR} > w^{DI}$; *(3) If* $v < v^B \equiv \dfrac{\alpha cL}{1 - L + \beta c}$, *then* $w^{DI} < w^{CB}(v)$, *otherwise* $w^{DI} \ge w^{CB}(v)$; *and, (4)* $w^{DI} > w^{CB}(1) > 0$.

*Proof.* See Appendix 8.7.

Lemma 4 implies that for liquidity shocks below $v^B$ the liquidity provision of the central banker is dominating that of the deposit insurer in term of social welfare. Otherwise it is in the inverse. Furthermore, we observe that the deposit insurer as lender of last resort is always dominated by the unconditional bailout rule.

Since only the liquidity shock $v$ is public information the policy maker will allocate the lender of last resort responsibilities conditional on the size of the liquidity shock to maximize the expected social welfare. Lemma 4 implies the following second-best optimal allocation:

**Proposition 7** *Assume that* $E\left(\tilde{u} \mid u \le u^{DI}\right) > u^*$. *It is optimal to allocate the lender of last resort responsibilities to the central banker for liquidity shortfalls below the threshold* $v^* \in (v^A, v^B)$. *Otherwise, it is socially optimal to apply the unconditional bailout rule.*

Proposition 7 shows that the introduction of the deposit insurer does not affect the allocation of responsibilities in the benchmark case. However, the existence of the deposit insurer sets an upper limit $v^B$ for the threshold $v^*$ where the responsibilities are transfered from the central banker to the unconditional bailout rule. Above $v^B$ the central banker is more strict and less in line with the first-best liquidity provision than the deposit insurer.

So the central banker's mandate should be restricted to $v^B$.

Furthermore, the deposit insurer sets the condition which insure the existence of the allocation of responsibilities defined in proposition 7. Ponce (2010) points out that the condition $E\left(\overset{\sim}{u} \mid u \leq u^{DI}\right) > u^*$ implies that the asset quality of a random bank is more likely to be of average quality (i.e. $u \in [u^*, u^{DI}]$) than of low quality (i.e. $u \in [0, u^*]$). Since in the interval $[u^*, u^{DI}]$ the deposit insurer will not provide socially desirable emergency loan, it is more likely that a socially desirable emergency loan is not provided compared to a socially non-desirable liquidity supports granted through the unconditional bailout rule for liquidity shortfalls in $u \in [0, u^*]$. For this reason, it is welfare-enhancing to apply the unconditional bailout rule instead of allocating lender of last resort responsibilities to the deposit insurer. For small liquidity shocks the central banker's lending decision is the closest to the first-best solution. When liquidity shocks increase the central banker becomes too tough, so that for small liquidity shocks the allocation of the lender of last resort responsibilities to the central banker is welfare enhancing.

## 6.2 Lender of last resort policy for the non-systemic bank

Suppose that the deposit insurer is the lender of last resort for the non-systemic bank. The deposit insurer will only support the non-systemic bank if the expected utility from providing liquidity is superior to the utility of closing the non-systemic bank. Due to the assumption $R - \gamma > 1$ the non-systemic bank's return is high enough to repay all deposits in case systemic bank fails. This implies that the deposit insurer's liquidity provision is independent of the systemic bank's state. If the deposit insurer provides the emergency loan $v_N$ the non-systemic bank will fail with probability of $(1 - u_N)$. It follows that the bank will be unable to repay $v_N$. The deposit insurer has to compensate the remaining depositors $1 - v_N$ and incurs the political cost of a bank failure $\beta c$. The expected utility of providing the liquidity is $(-(1 - u_N)(1 + \beta c))$. If the deposit insurer refuses the emergency loan the non-systemic bank will be closed and the deposit insurer will incur the political cost $\beta c$. In addition the deposit insurer has to compensate all depositors, but it can realize the liquidation value $L$. So the utility from closing the non-systemic bank is $L-1-\beta c$. The deposit insurer will lend the amount $v_N$ if:

$$-(1-u_N)(1+\beta c) \geq L-1-\beta c,$$

$$u_N \geq u_N^{DI} \equiv \frac{L}{1+\beta c}. \tag{38}$$

If the solvency signal is below $u_N^{DI}$ the deposit insurer will refuse the emergency loan. As in the benchmark case the threshold for the liquidity provision is independent of the liquidity shock. Comparing (38) with the threshold in the benchmark case in equation (36) shows that both lending decisions are equivalent.

Having defined the threshold on the solvency signal for the liquidity provision of the deposit insurer we derive from the social welfare function in (10) and the fist-best solvency signal thresholds in (11) and (12) the normalized expected social welfare function for the non-systemic bank given the deposit insurer is the lender of last resort:

$$w_N^{DI,\nu} = \int_{u_N^{DI}}^{1} (u_N - u_N^{\nu})\mathrm{d}F(u), \tag{39}$$

where $\nu \in \{SS, SF\}$ indicates the state of the systemic bank. Lemma 5 presents some properties of (39), the normalized expected social welfare functions of the central banker in equation (18) and the one of the unconditional bailout rule in equation (19).

**Lemma 5** *Assume that* $E\left(\tilde{u}_N \mid u_N \leq u_N^{DI}\right) > u_N^{SF}$. *Then, (1) if the systemic bank succeeded,* $\nu = SS$ *(respectively failed,* $\nu = SF$*), then (i)* $w_N^{CB,SS}(v_N)$ *is increasing in* $v_N$ *if* $v_N < v_N^A \equiv \frac{\alpha c L}{R-L+c}$ *(respectively* $v_N < v_N^C \equiv \frac{\alpha c L}{R-L+c-\gamma}$*), (ii) decreasing if* $v_N > v_N^A$ *(respectively* $v_N > v_N^C$*), and (iii) has a global maximum at* $v_N = v_N^A$ *(respectively at* $v_N = v_N^C$*); (2)* $w_N^{CB,\nu}(0) = w_N^{UBR,\nu} > w_N^{DI,\nu}$*; (3) If* $v_N < v_N^B \equiv \frac{\alpha c L}{1-L+\beta c}$*, then* $w_N^{DI,\nu} < w_N^{CB,\nu}(v_N)$*, otherwise* $w_N^{DI,\nu} \geq w_N^{CB,\nu}(v_N)$*; (4)* $w_N^{DI,\nu} > w_N^{CB,\nu}(1) > 0 \ \forall \ \nu \in \{SS, SF\}$.

*Proof.* See Appendix 8.8.

As for the benchmark case Lemma 5 implies that applying the unconditional bailout rule always dominates the deposit insurer as a lender of last resort. For liquidity shocks below $v_N^B$ the central banker as the lender of last resort yields a higher normalized social welfare than appointing the deposit insurer. If the liquidity shock is above above $v_N^B$ the order inverses and the normalized social welfare given the deposit insurer acts as the lender of last resort is higher.

The policy maker will allocate the lender of last resort responsibilities according to the size of the verifiable liquidity shock $v_N$ to maximize the expected social welfare. The following second-best optimal allocation results from lemma 5:

**Proposition 8** *Assume that* $E\left(\tilde{u}_N \mid u_N \le u_N^{DI}\right) > u_N^{SF}$. *If the systemic bank succeeded,* $v = SS$ *(respectively failed,* $v = SF$ *), there exists a threshold for the liquidity shortfall of the non-systemic bank* $v_N^{SS} \in (v_N^A, v_N^B)$ *(respectively* $v_N^{SF} \in (v_N^C, v_N^B)$ *) so that it is optimal to allocate the lender of last resort responsibilities for the non-systemic bank to the central banker for liquidity shortfalls below the threshold and to apply the unconditional bailout rule for liquidity shortfalls above it.*

The introduction of the deposit insurer into the model implements an upper limit $v_N^B$ for the threshold where the mandate for the lender of last resort responsibilities is handed over from the central banker to the unconditional bailout rule. This threshold is equivalent to the one in the benchmark case when the deposit insurer is considered. The existence of the deposit insurer affects the condition which ensure Proposition 8. Given condition $E\left(\tilde{u}_N \mid u_N \le u_N^{DI}\right) > u_N^{SF}$ it is more likely that a random non-systemic bank's asset is of average quality (i.e. $u \in [u_N^{SF}, u_N^{DI}]$) than of low quality (i.e. $u \in [0, u_N^{SF}]$). It is more likely that a non-systemic bank for which liquidity support is socially optimal does not receive an emergency loan from the deposit insurer than that a non-systemic bank with low quality asset is bailed out unconditionally. Therefore, the policy maker chooses to apply the unconditional bail out rule instead of assigning the deposit insurer with the LLR responsibility.

It is not always optimal to support illiquid banks. For small liquidity shocks the central banker's threshold is closer to the socially optimal one than the unconditional bailout rule. For this reason the LLR responsibilities for small liquidity shocks are allocated to the central banker. If the liquidity shock is large the central banker's lending decision is too restrictive. The unconditional bailout rule maximizes the expected social welfare.

As stated in proposition 8 the existence of the deposit insurer does not change the allocation of the lender of last resort responsibilities. We can show that the extended set of policy instruments does not affect the responsibilities of the central banker given the state of the systemic bank. Proposition 9 summaries the central banker's mandate in both cases.

**Proposition 9** *The central banker should act as a lender of last resort in a larger range of liquidity shortfalls of the non-systemic bank when the systemic bank failed than when it succeeded (i.e.* $v_N^{SS} < v_N^{SF}$ *).*

*Proof.* See Appendix 8.9.

The intuition for this result is equivalent to the explanation when only the central banker and the unconditional bailout rule are considered. With a failure of the systemic bank the expected return of the non-systemic bank falls so that for all liquidity shocks the socially optimal threshold for the provision of the emergency loan increases. Since the central banker becomes less forbearing with increasing liquidity shocks the socially optimal lending decision is closer to the central banker's one for a larger interval of liquidity shocks. However, for very large liquidity shocks the central banker is still too stringent so that unconditional bail out rule maximizes the expected social welfare in this interval.

## 6.3 Lender of last resort policy for the systemic bank

Suppose now that the deposit insurer is the lender of last resort for the systemic bank. The deposit insurer's expected utility from the lender of last resort activities for the systemic bank is given by:

$$D_S = E\{1_S[-(1-u_S)(1+\beta c)+D_N^{SC}]+(1-1_S)[L-1-\beta c+D_N^{SL}]\},$$

$$D_S = E\{1_S[u_S(1+\beta c)-L+(D_N^{SC}-D_N^{SL})]+L-1-\beta c+D_N^{SL}\}. \tag{40}$$

If the deposit insurer provides the emergency loan the systemic bank will be successful with a probability $u_S$ and repays the liquidity assistance so that the deposit insurer does not suffer any losses. With a probability of $(1-u_S)$ the systemic bank fails. The deposit insurer loses the provided emergency loan $v_S$ and has to compensate the remaining depositors $(1-v_S)$. Additionally, the deposit insurer incurs the political cost $\beta c$. The decision to support the systemic bank influences the profitability of the non-systemic bank. Consequently the utility for the deposit insurer from the non-systemic bank given the systemic bank continues to operate $D_N^{SC}$ enters into the expected utility of the deposit insurer. In case the systemic bank is not supported the expected costs are $L-1-\beta c+D_N^{SL}$ where $L$ is the liquidation value of the systemic bank and $D_N^{SL}$ is the deposit insurer's utility from the non-systemic bank in case the systemic bank is closed.

According to proposition 8 the deposit insurer is not responsible for the provision of emergency loans to the non-systemic bank. For this reason it does not bear any political cost in case the non-systemic bank fails or is closed. However, the deposit insurer still has to compensate the non-systemic bank's depositors in case of distress. When the central banker is the lender of last resort and the non-systemic bank is not supported the deposit insurer has to compensate all depositors net of the liquidation value $(1-L)$. If the central banker provides the emergency loan but the non-systemic bank fails the deposit insurer only has to compensate the remaining depositors because the emergency loan from the central banker is not insured by the deposit insurance. Thus, the expected cost for the deposit insurer is $(1-u_N)(1-v_N)$. If the unconditional bailout rule is applied the deposit insurer has to compensate all depositors in case of a non-systemic bank's failure. Therefore, the expected cost is $(1-u_N)$. This said the utility for the deposit insurer from the non-systemic bank if the systemic bank S is not liquidated corresponds to:

$$D_N^{SC} = u_S\left[\int_0^{v_N^{SS}}\left[\int_0^{u_N^{CB}(v_N)} -(1-L)dF(u)+\int_{u_N^{CB}(v_N)}^{1} -(1-u_N)(1-v_N)dF(u)\right]dG(v_N)\right. \tag{41}$$

$$+\int_{v_N^{SS}}^{1}\int_0^1 -(1-u_N)(1-v_N)dF(u_N)dG(v_N)\Bigg]+(1-u_S)$$

$$\left[\int_0^{v_N^{SF}}\left[\int_0^{u_N^{CB}(v_N)}-(1-L)dF(u)+\int_{u_N^{CB}(v_N)}^{1}-(1-u_N)(1-v_N)(1-v_N)dF(u)\right]dG(v_N)\right.$$

$$\left.+\int_{v_N^{SF}}^{1}\int_0^1 -(1-u_N)dF(u)dG(v_N)\right],$$

where $v_N^{SS}$ and $v_N^{SS}$ are the second-best threshold as defined in proposition 8.

The deposit insurer's utility from the non-systemic bank if the systemic bank is liquidated is equal to:

$$D_N^{SL}=\int_0^{v_N^{SF}}\left[\int_0^{u_N^{CB}(v_N)}-(1-L)dF(u)+\int_{u_N^{CB}(v_N)}^{1}-(1-u_N)(1-v_N)dF(u)\right]dG(v_N) \quad (42)$$

$$+\int_{v_N^{SF}}^{1}\int_0^1 -(1-u_N)(1-v_N)dF(u)dG(v_N),$$

which follows the same reasoning as above. In case the central banker does not support bank N the deposit insurer liquidates the non-systemic bank and compensates all depositors. In this situation the expected cost for deposit insurer is equal to $-(1-L)$. If the central banker provided the emergency loan but bank N fails the deposit insurer has to compensate only the remaining depositors so that the expected costs are $-(1-u_N)(1-v_N)$. If the unconditional bailout rule is applied and bank N fails the deposit insurer compensates all depositors which leads to expected costs equal to $-(1-u_N)(1-v_N)$.

We define:

$$D_N^{\Delta}=\int_{v_N^{SS}}^{v_N^{SF}}\left[\int_0^{u_N^{CB}(v_N)}(1-L)-(1-u_N)(1-v_N)dF(u)\right]dG(v_N),$$

so that

$$D_N^{SC}-D_N^{SL}=u_S D_N^{\Delta}. \quad (43)$$

Given (40) and (43) the deposit insurer lends to bank S if:

$$u_S(1+\beta c+D_N^{\Delta})\geq L,$$

$$u_S\geq u_S^{DI}\equiv\frac{L}{1+\beta c+D_N^{\Delta}}, \quad (44)$$

where $D_N^{\Delta}$ represents the impact of the deposit insurer's behavior towards systemic bank on the expected cost related to the non-systemic bank. If the solvency signal is below $u_S^{DI}$ the deposit insurer will not provide the emergency loan and the systemic bank will be closed.

Using the solvency threshold of the deposit insurer defined in (44), the social welfare function from equation (20) and $u_S^* = \dfrac{L}{R + c + W_N^\Delta}$ we can derive the normalized expected social welfare function given the deposit insurer is the lender of last resort for the systemic bank:

$$w_S^{DI} = \int_{u_S^{DI}}^{1} (u_S - u_S^*) \mathrm{d}F(u). \tag{45}$$

Lemma 6 proves some properties of (45) and relates it with the normalized expected social welfare function for the systemic bank given the central banker is the lender of last resort or the unconditional bailout rule is applied:

**Lemma 6** *Assume that* $E\left(\tilde{u}_S \mid u_S \le u_S^{DI}\right) > u_S^*$. *Then, (1)(i)* $w_S^{CB}(v_S)$ *is increasing in* $v_S$ *if* $v_S < v_S^A \equiv \dfrac{L(\alpha c + B_N^\Delta)}{R - L + c + W_N^\Delta}$, *(ii) decreasing if* $v_S > v_S^A$, *and (iii) has a global maximum at* $v_S = v_S^A$; *(2)* $w_S^{CB}(0) = w_S^{UBR} > w_S^{DI}$; *(3) If* $v_S < v_S^B \equiv \dfrac{L(\alpha c + B_N^\Delta)}{1 - L + \beta c + D_N^\Delta}$, *then* $w_S^{DI} < w_S^{CB}(v_S)$, *otherwise* $w_S^{DI} \ge w_S^{CB}(v_S)$; *(4)* $w_S^{DI} > w_S^{CB}(1) > 0$.

*Proof.* See Appendix 8.10.

Lemma 6 shows that the normalized social welfare from the allocation of the lender of last resort responsibilities to the deposit insurer is dominated by the unconditional bailout rule and for liquidity shocks below $v_S^B$ also by the central banker. If the liquidity shocks are above $v_S^B$ the normalized social welfare if the deposit insurer is the lender of last resort exceeds the normalized social welfare when the central banker acts as the lender of last resort.

The policy maker will allocate the lender of last resort responsibilities between the central banker, the deposit insurer and the unconditional bailout rule conditional on the liquidity shortfall in order to maximize the normalized social welfare. The following proposition is derived from the properties proven in Lemma 6:

**Proposition 10** *Assume* $E\left(\tilde{u}_S \mid u_S \leq u_S^{DI}\right) > u_S^*$. *There exist a liquidity shortfall* $v_S^* \in \{v_S^A, v_S^B\}$ *so that it is optimal to allocate the lender of last resort responsibilities to the central banker for all liquidity shock smaller than* $v_S^*$. *Above* $v_S^*$ *the it is optimal to apply the unconditional bailout rule.*

The extended set of policy instruments does not affect the allocation of lender of last resort responsibilities for the systemic bank. It provides however an upper limit $v_S^B$ for the threshold where the mandate is handed over from the central banker to the unconditional bailout rule because from a social welfare point of view the central banker's lending decision for $v_S > v_S^B$ is worse than the deposit insurer's liquidity provision.

As for the benchmark case and the non-systemic bank the deposit insurer determines the condition ensuring proposition 10. Condition $E\left(\tilde{u}_S \mid u_S \leq u_S^{DI}\right) > u_S^*$ implies that the asset of a random systemic bank is more likely to be of an average quality (i.e. $u \in [u_S^*, u_S^{DI}]$) than of low quality (i.e. $u \in [0, u_S^*]$). For this reason it is more likely that the deposit insurer does not provide the socially optimal emergency loan to a systemic bank than that a systemic bank with low quality assets is bailed out unconditionally. Therefore, the policy maker chooses to apply the unconditional bail out rule instead of assigning the deposit insurer with the LLR responsibility.

As above it is not always optimal to support illiquid banks. For small liquidity shock the central banker's threshold is closer to the socially optimal one than the unconditional bailout rule. Therefore, the LLR responsibilities for small liquidity shocks are allocated to the central banker. If the liquidity shock is large the central banker's lending decision is too restrictive. The unconditional bailout rule maximizes the expected social welfare.

# 7 Conclusion

This paper analyses the optimal institutional allocation of lender of last resort responsibilities in a framework with a systemic and a non-systemic bank. The failure of the systemic bank hurts the return of the non-systemic bank but not vise-versa. Both banks are exposed to a liquidity shock. Taking for granted that other source of external funding are not available, public intervention by a lender of last resort is necessary to avoid socially inefficient and detrimental bank failures.

We show that the lender of last resort responsibilities should be shared between the central bank and an unconditional bailout rule where banks receive emergency liquidity assistance regardless of their solvency. The intuition for this result is as follows. On the one hand, the unconditional bailout rule too often provides socially undesirable emergency loans. For small liquidity shocks the central bank can improve expected social welfare because its emergency liquidity assistance is conditional on the bank's solvency. On the other hand the central bank becomes more restrictive with increasing liquidity shortfalls. The central bank might even refuse to provide socially optimal emergency liquidity assistance if the required emergency loan is too large. For this reason the unconditional bailout rule should be applied for large liquidity shortfalls.

We find that the allocation of lender of last resort responsibilities for the non-systemic bank should be conditional on the state of the systemic bank. Given the negative impact of a systemic bank's failure on the profitability of the non-systemic bank the central bank should be given more responsibilities in the event that the systemic bank collapses.

For the systemic bank however the determination of the range of action for the central bank and the unconditional bailout rule is ambiguous because there are counteracting effects. On the one hand, from the social optimum point of view more forbearance for systemic bank is desirable so that the central bank should have less lender of last resort responsibilities. On the other hand, the central bank itself will be less restrictive in order to limit its potential losses for the non-systemic bank. This leads to more responsibilities for the central bank and a smaller range where the unconditional bailout rule should be applied.

# 8 Appendix

## 8.1 Proof of Lemma 1

(1) The first derivative of $w^{CB}(v)$ is: $\dot{w}^{CB}(v) = -\dot{u}^{CB}(v)\left[u^{CB}(v) - u^*\right]f(u)$, where $f$ is the density function of the random variable $\overset{\frown}{u}$. Since $\dot{u}^{CB}(v)$ and $f(u)$ are positive for all $v$ and $u$, $w^{CB}(v)$ is increasing in $v$ if $u^{CB}(v) < u^*$, decreasing if $u^{CB}(v) > u^*$, and has a global maximum for $u^{CB}(v) = u^*$. Since $\dot{u}^{CB}(v) > 0$ and $v^A$ is so that $u^{CB}(v^A) = u^*$ (see Figure 2), the result follows.

(2) Since $u^{CB}(0) = 0 = u^{UBR}$, then $w^{CB}(0) = w^{UBR}$.

(3)(a) Assume $w^{CB}(0) - w^{CB}(1) \le 0$. Then $\int_{u^{CB}(0)}^{1}(u - u^*)dF(u) - \int_{u^{CB}(1)}^{1}(u - u^*)dF(u) \le 0$, $\int_{u^{CB}(0)}^{u^{CB}(1)}(u - u^*)dF(u) \le 0$, $\left[E\left(\tilde{u} \mid u^{CB}(0) \le u \le u^{CB}(1)\right) - u^*\right]\left[F\left(u^{CB}(1)\right) - F\left(u^{CB}(0)\right)\right] \le 0$, and $E\left(\tilde{u} \mid u^{CB}(0) \le u \le u^{CB}(1)\right) \le u^*$. A contradiction.

(b) $w^{CB}(1) = \int_{u^{CB}(1)}^{1}(u - u^*)dF(u) = \left[E\left(\tilde{u} \mid u > u^{CB}(1)\right) - u^*\right]\left[1 - F\left(u^{CB}(1)\right)\right]$. Since $u^* = \dfrac{L}{R+c} < \dfrac{1}{1+\alpha c} = u^{CB}(1) < 1$ both factors are positive, then $w_{CB}(1) > 0$.

## 8.2 Proof of Proposition 2

(1) $u^* = u_N^{SS} < u_N^{SF}$ because $\gamma > 0$ (2) $u^{CB} = u_N^{CB}$ (3) $u^{UBR} = u_N^{UBR}$

## 8.3 Proof of Lemma 2

(1) The first derivative of $w_N^{CB,v}(v_N)$ is: $\dot{w}_N^{CB,v}(v_N) = -\dot{u}_N^{CB}(v_N)\left[u_N^{CB}(v_N) - u_N^{v}\right]f(u)$,

where $f$ is the density function of the random variable $\widetilde{u_N}$. Since $\dot{u}_N^{CB}(v_N)$ and $f(u)$ are positive for all $v_N$ and $u_N$, $w_N^{CB,\nu}(v_N)$ is increasing in $v_N$ if $u_N^{CB}(v_N) < u_N^{\nu}$, decreasing if $u_N^{CB}(v_N) > u_N^{\nu}$, and has a global maximum for $u_N^{CB}(v_N) = u_N^{\nu}$. Since $\dot{u}_N^{CB}(v_N) > 0$ and $v_N^A$ (respectively $v_N^C$) is so that $u_N^{CB}(v_N^A) = u_N^{SS}$ (respectively $u_N^{CB}(v_N^C) = u_N^{SF}$) (see Figure 3), the result follows.

(2) Since $u_N^{CB}(0) = 0 = u_N^{UBR}$, then $w_N^{CB,\nu}(0) = w_N^{UBR,\nu}$.

(3)(a) Assume $w_N^{CB,\nu}(0) - w_N^{CB,\nu}(1) \leq 0$. Then $\int_{u_N^{CB,\nu}(0)}^{1}(u_N - u_N^{\nu})dF(u) - \int_{u_N^{CB,\nu}(1)}^{1}(u_N - u_N^{\nu})dF(u) \leq 0$, $\int_{u_N^{CB,\nu}(0)}^{u_N^{CB,\nu}(1)}(u_N - u_N^{\nu})dF(u) \leq 0$, $\left[E\left(\tilde{u}_N \mid u_N \leq u_N^{CB,\nu}(1)\right) - u_N^{\nu}\right]\left[F\left(u_N^{CB,\nu}(1)\right) - F\left(u_N^{CB,\nu}(0)\right)\right] \leq 0$, and $E\left(\tilde{u}_N \mid u_N \leq u_N^{CB,\nu}(1)\right) \leq u_N^{\nu}$. A contradiction.

$$\text{(b) } w_N^{CB}(1) = \int_{u_N^{CB,\nu}(1)}^{1}(u_N - u_N^{\nu})dF(u) = \left[E\left(\tilde{u}_N \mid u_N > u_N^{CB}(1)\right) - u_N^{\nu}\right]\left[1 - F\left(u_N^{CB,\nu}(1)\right)\right].$$

Since $u_N^{\nu} < \dfrac{1}{1+\alpha c} = u_N^{CB}(1) < 1$ both factors are positive, then $w_N^{CB}(1) > 0$.

## 8.4 Proof of Proposition 4

Given equation (18) and (19) (a) $w_N^{UBR,SS} - w_N^{UBR,SF} = u_N^{SF} - u_N^{SS}$, (b) $w_N^{CB,SS}(v_N) - w_N^{CB,SF}(v_N) = (u_N^{SF} - u_N^{SS})[1 - F(u_N^{CB}(v_N))]$, (c) $w_N^{CB,SS}(v_N) - w_N^{CB,SF}(v_N)$ is non-increasing in $v_N$, (d) To the right of $v_N^C$ both $w_N^{CB,SS}(v_N)$ and $w_N^{CB,SF}(v_N)$ are decreasing. It follows that $v_N^{SS} < v_N^{SF}$.

## 8.5 Proof of Proposition 5

The minimum solvency requirement in the first-best is $u_S^* \equiv \dfrac{L}{R + c + W_N^{\Delta}}$ where

$W_N^\Delta = E\{(1_N^{SS} - 1_N^{SF})(u_N(R+c) - L) + 1_N^{SF} u_N \gamma\} \geq 0$. It follows that $u_S^* \leq u^*$.

## 8.6 Proof of Lemma 3

(1) The first derivative of $w_S^{CB}(v_S)$ is: $\dot{w}_S^{CB}(v_S) = -\dot{u}_S^{CB}(v_S)\left[u_S^{CB}(v_S) - u_S^*\right]f(u)$, where $f$ is the density function of the random variable $\tilde{u}_S$. Since $\dot{u}_S^{CB}(v_S)$ and $f(u)$ are positive for all $v_S$ and $u_S$, $w_S^{CB}(v_S)$ is increasing in $v_S$ if $u_S^{CB}(v_S) < u_S^*$, decreasing if $u_S^{CB}(v_S) > u_S^*$, and has a global maximum for $u_S^{CB}(v_S) = u_S^*$. Since $\dot{u}_S^{CB}(v_S) > 0$ and $v_S^A$ is such that $u_S^{CB}(v_S^A) = u_S^*$, the result follows.

(2) Since $u_S^{CB}(0) = 0 = u_S^{UBR}$, then $w_S^{CB}(0) = w_S^{UBR}$.

(3)(a) Assume $w_S^{CB}(0) - w_S^{CB}(1) \leq 0$. Then $\int_{u_S^{CB}(0)}^1 (u_S - u_S^*) dF(u) - \int_{u_S^{CB}(1)}^1 (u_S - u_S^*) dF(u) \leq 0$, $\int_{u_S^{CB}(0)}^{u_S^{CB}(1)} (u_S - u_S^*) dF(u) \leq 0$,

$$\left[E\left(\tilde{u}_S \mid u_S^{CB}(0) \leq u_S \leq u^{CB}(1)\right) - u_S^*\right]\left[F\left(u_S^{CB}(1)\right) - F\left(u_S^{CB}(0)\right)\right] \leq 0,$$

and $E\left(\tilde{u}_S \mid u_S \leq u_S^{CB}(1)\right) \leq u_S^*$. A contradiction. Together with property (2) this imply $w_S^{UBR} > w_S^{CB}$.

(b) $w_S^{CB}(1) = \int_{u_S^{CB}(1)}^1 (u_S - u_S^*) dF(u) = \left[E\left(\tilde{u}_S \mid u_S > u_S^{CB}(1)\right) - u_S^*\right]\left[1 - F\left(u_S^{CB}(1)\right)\right]$. Since $u_S^{CB}(1) = \dfrac{1}{1 + \alpha c + B_N^\Delta} < 1$ and assumption $E\left(\tilde{u}_S \mid u_S \leq u_S^{CB}(1)\right) > u_S^*$ implies

$E\left(\tilde{u}_S \mid u_S \geq u_S^{CB}(1)\right) > u_S^*$ both factors are positive, then $w_S^{CB}(1) > 0$.

## 8.7 Proof of Lemma 4

The proof of lemma 4 is taken from Ponce10 .

(1) The first derivative of $w^{CB}(v)$ is: $\dot{w}^{CB}(v) = -\dot{u}^{CB}(v)\left[u^{CB}(v) - u^*\right]f(u)$, where $f$ is the density function of the random variable $\tilde{u}$ . Since $\dot{u}^{CB}(v)$ and $f(u)$ are positive for all $v$ and $u$, $w^{CB}(v)$ is increasing in $v$ if $u^{CB}(v) < u^*$, decreasing if $u^{CB}(v) > u^*$, and has a global maximum for $u^{CB}(v) = u^*$. Since $\dot{u}^{CB}(v) > 0$ and $v^A$ is such that $u^{CB}(v^A) = u^*$, the result follows.

(2) (a) Since $u^{CB}(0) = 0 = u^{UBR}$, then $w^{CB}(0) = w^{UBR}$. (b) Assume $w^{UBR} - w^{DI} \le 0$. Then $\int_0^1 (u - u^*)dF(u) - \int_{u^{DI}}^1 (u - u^*)dF(u) \le 0$, $\int_0^{u^{DI}} (u - u^*)dF(u) \le 0$, $\left[E\left(\tilde{u} \mid u \le u^{DI}\right) - u^*\right]F\left(u^{DI}\right) \le 0$, and $E\left(\tilde{u} \mid u \le u^{DI}\right) \le u^*$. A contradiction.

(3) Since $v^B$ is so that $u^{CB}(v^B) = u^{DI}$, then $w^{CB}(v^B) = w^{DI}$. Properties 1 and 2 imply that $w^{DI} < w^{UBR} \le w^{CB}(v)$ for $v < v^B$ and that $w^{DI} \ge w^{CB}(v)$ for $v \ge v^B$.

(4) Since $v^B < 1$, property 3 implies that $w^{DI} > w^{CB}(1)$. $w^{CB}(1) = \int_{u^{CB}(1)}^1 (u - u^*)dF(u) = \left[E\left(\tilde{u} \mid u > u^{CB}(1)\right) - u^*\right]\left[1 - F\left(u^{CB}(1)\right)\right]$. Since $u^* = \dfrac{L}{R+c} < \dfrac{1}{1+\alpha c} = u^{CB}(1) < 1$ both factors are positive, then $w_{CB}(1) > 0$.

## 8.8 Proof of Lemma 5

(1) The first derivative of $w_N^{CB,v}(v_N)$ is: $\dot{w}_N^{CB,v}(v_N) = -\dot{u}_N^{CB}(v_N)\left[u_N^{CB}(v_N) - u_N^v\right]f(u)$, where $f$ is the density function of the random variable $\tilde{u}_N$ . Since $\dot{u}_N^{CB}(v_N)$ and $f(u)$ are positive for all $v_N$ and $u_N$, $w_N^{CB,v}(v_N)$ is increasing in $v_N$ if $u_N^{CB}(v_N) < u_N^v$, decreasing if $u_N^{CB}(v_N) > u_N^v$, and has a global maximum for $u_N^{CB}(v_N) = u_N^v$. Since $\dot{u}_N^{CB}(v_N) > 0$ and $v_N^A$ (respectively $v_N^C$) is so that $u_N^{CB}(v_N^A) = u_N^{SS}$ (respectively $u_N^{CB}(v_N^C) = u_N^{SF}$), the result follows.

(2) (a) Since $u_N^{CB}(0) = 0 = u_N^{UBR}$, then $w_N^{CB,\nu}(0) = w_N^{UBR,\nu}$. (b) Assume $w_N^{UBR,\nu} - w_N^{DI,\nu} \leq 0$. Then $\int_0^1 (u_N - u_N^{\nu}) dF(u) - \int_{u_N^{DI}}^1 (u_N - u_N^{\nu}) dF(u) \leq 0$,

$\int_0^{u_N^{DI}} (u_N - u_N^{\nu}) dF(u) \leq 0$, $\left[ E\left( \overset{\sim}{u}_N \mid u_N \leq u_N^{DI} \right) - u_N^{\nu} \right] F(u_N^{DI}) \leq 0$, and

$E\left( \overset{\sim}{u}_N \mid u_N \leq u_N^{DI} \right) \leq u_N^{\nu}$. A contradiction.

(3) Since $v_N^B$ is so that $u_N^{CB}(v_N^B) = u_N^{DI}$, then $w_N^{CB,\nu}(v_N^B) = w_N^{DI,\nu}$. Properties 1 and 2a imply that $w_N^{DI,\nu} < w_N^{UBR,\nu} \leq w_N^{CB,\nu}(v_N)$ for $v_N < v_N^B$ and that $w_N^{DI,\nu} \geq w_N^{CB,\nu}(v_N)$ for $v_N \geq v_N^B$.

(4) Since $v_N^B < 1$, property 3 implies that $w_N^{DI,\nu} > w_N^{CB,\nu}(1)$. $w_N^{CB}(1) = \int_{u_N^{CB,\nu}(1)}^1 (u_N - u_N^{\nu}) dF(u) = \left[ E\left( \overset{\sim}{u}_N \mid u_N > u_N^{CB}(1) \right) - u_N^{\nu} \right] \left[1 - F\left(u_N^{CB,\nu}(1)\right)\right]$. Since

$u_N^{\nu} < \dfrac{1}{1+\alpha c} = u_N^{CB}(1) < 1$ both factors are positive, then $w_N^{CB}(1) > 0$.

## 8.9 Proof of Proposition 9

Given equation (18) and (19) (a) $w_N^{UBR,SS} - w_N^{UBR,SF} = u_N^{SF} - u_N^{SS}$, (b) $w_N^{CB,SS}(v_N) - w_N^{CB,SF}(v_N) = (u_N^{SF} - u_N^{SS})[1 - F(u_N^{CB}(v_N))]$, (c) $w_N^{CB,SS}(v_N) - w_N^{CB,SF}(v_N)$ is non-increasing in $v_N$, (d) To the right of $v_N^C$ both $w_N^{CB,SS}(v_N)$ and $w_N^{CB,SF}(v_N)$ are decreasing. It follows that $v_N^{SS} < v_N^{SF}$.

## 8.10 Proof of Lemma 6

(1) The first derivative of $w_S^{CB}(v_S)$ is: $\dot{w}_S^{CB}(v_S) = -\dot{u}_S^{CB}(v_S)\left[u_S^{CB}(v_S) - u_S^{\nu}\right] f(u)$, where $f$ is the density function of the random variable $\overset{\sim}{u}_S$. Since $\dot{u}_S^{CB}(v_S)$ and $f(u)$ are positive for all $v_S$ and $u_S$, $w_S^{CB}(v_S)$ is increasing in $v_S$ if $u_S^{CB}(v_S) < u_S^{\nu}$, decreasing if $u_S^{CB}(v_S) > u_S^{\nu}$, and has a global maximum for $u_S^{CB}(v_S) = u_S^{\nu}$. Since $\dot{u}_S^{CB}(v_S) > 0$ and $v_S^A$ is so that

$u_S^{CB}\left(v_S^A\right)=u_S^*$, the result follows.

(2) (a) Since $u_S^{CB}(0)=0=u_S^{UBR}$, then $w_S^{CB}(0)=w_S^{UBR}$. (b) Assume $w_S^{UBR}-w_S^{DI}\leq 0$. Then $\int_0^1\left(u_S-u_S^*\right)dF(u)-\int_{u_S^{DI}}^1\left(u_S-u_S^v\right)dF(u)\leq 0$, $\int_0^{u_S^{DI}}\left(u_S-u_S^*\right)dF(u)\leq 0$,

$\left[E\left(\tilde{u}_S \mid u_S\leq u_S^{DI}\right)-u_S^*\right]F\left(u_S^{DI}\right)\leq 0$, and $E\left(\tilde{u}_S \mid u_S\leq u_S^{DI}\right)\leq u_S^*$. A contradiction.

(3) Since $v_S^B$ is so that $u_S^{CB}\left(v_S^B\right)=u_S^{DI}$, then $w_S^{CB}\left(v_S^B\right)=w_S^{DI}$. Properties 1 and 2a imply that $w_S^{DI}<w_S^{UBR}\leq w_S^{CB}\left(v_S\right)$ for $v_S<v_S^B$ and that $w_S^{DI}\geq w_S^{CB}\left(v_S\right)$ for $v_S\geq v_S^B$.

(4) Since $v_S^B<1$, property 3 implies that $w_S^{DI}>w_S^{CB}(1)$. $w_S^{CB}(1)=\int_{u_S^{CB}(1)}^1\left(u_S-u_S^*\right)dF(u)=\left[E\left(\tilde{u}_S \mid u_S>u_S^{CB}(1)\right)-u_S^*\right]\left[1-F\left(u_S^{CB}(1)\right)\right]$. Since

$u_S^*=\dfrac{L}{R+c+W_N^\Delta}<\dfrac{1}{1+\alpha c+B_N^\Delta}=u_S^{CB}(1)<1$ both factors are positive, then $w_S^{CB}(1)>0$.

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