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The relation between the level and uncertainty of inflation

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RESUMEN

En este trabajo se analizan las implicaciones que tiene la presencia de observaciones atípicas y heteroscedasticidad condicional en series temporales con características similares a las observadas en las series mensuales de IPC de los países del G-7. Se realizan estimaciones del nivel y la volatilidad de la inflación para estas economías y se discuten algunos de los problemas que presenta la investigación aplicada de la relación entre el nivel y la volatilidad de la inflación. Los resultados empíricos indican que en la mayor parte de las series de IPC del G-7 se detecta simultáneamente la presencia de observaciones atípicas y heteroscedasticidad condicional y que las estimaciones de volatilidad condicional son sensibles a la presencia de observaciones atípicas. Se observa que la dependencia temporal encontrada en la varianza condicional es duradera.

ABSTRACT

This paper focus on the problems faced in the empirical investigation of the relation between the level and volatility of inflation. Monthly inflation series seem to be affected by both the presence of outliers and conditional heteroscedasticity. First, the paper illustrates the implications that the presence of outliers and conditional heteroscedasticity have on the usual residual diagnostics. Then, estimates of the level and volatility of inflation are obtained for each of the countries of the G-7 group. Empirical evidence for the majority of the inflation series for these countries indicates both the presence of outliers and conditional heteroscedasticity, and that estimates of the latter are sensitive to the presence of outliers. Finally, the temporal dependence found in the conditional variance is enduring.

Key Words: Conditional Heteroscedasticity, Diagnostic, Inflation, Outlier, Stochastic Volatility.

JEL: E31.

1. INTRODUCTION

Economists often argued that there is a positive relationship between inflation and uncertainty about future inflation; see, for example, Okun (1971), Friedman (1977), and Ball (1992). Testing this hypothesis involves, first of all, testing for the temporal evolution of uncertainty. In this sense, most empirical studies, using US inflation data, find that uncertainty, measured by the conditional variance, evolves over time. However, these studies reach different results when analyzing the Friedman hypothesis. Engle (1983) and Bollerslev (1986) fitted ARCH (AutoRegressive Conditional Heteroscedasticity) and GARCH (Generalized ARCH) models respectively to US inflation and concluded that, although the conditional variance of inflation evolves over time, there is no relation between the level and future uncertainty of inflation. In a related paper, Cosimano and Jansen (1988) found little evidence of a link between the level and uncertainty when the level of inflation is high and none when it is low. In subsequent papers, Ball and Cecchetti (1990) and Evans (1991) found evidence of the Friedman hypothesis in the long run. Brunner and Hess (1993), using a state-dependent model with asymmetric relationships, also found strong evidence of the inflation-uncertainty link. Kim (1993) considering regime shifts in both the level and variance of inflation find that higher inflation is associated with higher long-run uncertainty. Finally, Baillie et al. (1996) analyzing monthly inflation series for ten countries found that for three high inflation economies (Argentina, Brazil, and Israel) and the United Kingdom there is evidence that the mean and volatility of inflation interact.

In order to test properly the Friedman hypothesis it is fundamental to consider carefully the econometric methodology used. It is well known that inflation series are often affected by the presence of outliers that may affect substantially the modeling of their dynamics. Lorenzo (1997), analyzing the dynamic properties of monthly series of inflation of the G7 countries (Germany, Canada, United States, France, Italy, Japan and United Kingdom), found outliers in all of them. However, none of the previous papers take into account the potential presence of outliers in time series of inflation and how these outliers can influence the estimated models of both the conditional mean and conditional variance. Furthermore, usual methods for outlier identification are based on the assumption of homoscedasticity; see, for example, Chen *et al*. (1991) and Chen and Liu (1993). If conditional variances evolve over time, observations corresponding to periods when these variances are high (over the constant marginal variance), may have higher probability of being identified as outliers are. In sum, it seems worthwhile to account for the presence of both outliers and conditional heteroscedasticity when analyzing time series of inflation. Both effects have rather different economic interpretations and, consequently, it is fundamental to properly separate them. Furthermore, some authors argue that conditional heteroscedasticity is often found in macroeconomic time series because of the presence of outliers. For example, Balke and Fomby (1994) after analyzing several US macroeconomic series show that, with the exception of inflation, controlling for outliers eliminates most of the evidence of non-linearity found in the raw series. They also present evidence of clustering of outliers across time, which could be confused with heteroscedasticity. They argue that conditional heteroscedasticity models are parsimonious characterizations of the large shocks hypothesis, but that outliers may be a better characterization of the data than conditional heteroscedasticity.

The objective of this paper is twofold. First, we are interested in illustrating the econometric problem faced when outliers and conditional heteroscedasticity appear together in a time series. Notice that we are not proposing a methodology to handle this problem but pointing out the problems faced when modeling uncertainty in the presence of outliers. Secondly, we want to analyze the existence of the inflation-uncertainty relationship. The analysis will be based on univariate seasonal ARIMA models with interventions and heteroscedastic disturbances. We are modeling conditional heteroscedasticity using stochastic volatility models so we can obtain smoothed estimates of volatility which can be compared with estimated levels of inflation.

The rest of the paper is organized as follows. In section 2 we simulate several time series to analyze how the presence of outliers and conditional heteroscedasticity interact and can be confused when looking at the usual residual diagnostic statistics. First, we show how the presence of outliers in a time series may affect the diagnostics of residuals from ARIMA models and can be missed with conditional heteroscedasticity. On the other hand, conditional heteroscedasticity can generate what can be identified by traditional procedures as outliers. Finally, if both outliers and conditional heteroscedasticity appear together in a series, it will be difficult to identify and separate properly both effects. In section 3, we analyze monthly series of inflation for the G7 countries. First, we fit seasonal ARIMA models with intervention analysis for the conditional mean. Then, we consider the estimation of the conditional variance of the residuals. In section 4, we analyze the causal relationship between the level and uncertainty of inflation. Finally, section 5 contains the conclusions.

2. RELATION BETWEEN OUTLIERS AND CONDITIONAL HETEROSCEDASTICITY

Conditional heteroscedastic time series are characterized by having slowly decaying autocorrelations in the squared observations and non-normal distributions with excess kurtosis; see, for example, Bollerslev et al. (1994) and Gyshels et al. (1996). Therefore, tests for evolving conditional variances are usually based on sample autocorrelations of squared residuals and tests of non-normality. On the other hand, outliers may also cause non-normal distributions with excess kurtosis and, if they appear in clusters, autocorrelations of squares. Consequently, it is of interest to analyze how the presence of outliers in a time series may be confused with conditional heteroscedasticity when carrying out diagnostics on the residuals. In this sense, using asymptotic arguments, van Dijk et al. (1996) show that the bias in estimating the conditional mean in presence of outliers, adversely affects both the size and power properties of the standard Lagrange Multiplier (LM) test for ARCH. This test was originally proposed by Engle (1982) and it is asymptotically equivalent to the McLeod and Li (1983) test based on the autocorrelations of squared observations. They show that the test rejects the null hypothesis of homoscedasticity too often when it is in fact true, while having difficulty detecting genuine ARCH effects. Finally, they proposed a robust test, which has better size and power properties than the standard test when the proportion of outliers is low (1%). However, the size and power of the robust test are also badly affected by the presence of outliers when the proportion is 5%.

On the other hand, conditional heteroscedasticity may generate what may be confused with outliers. Traditional methods for detecting outliers are based on a constant variance; see, for example, Chen *et al*. (1991). However, if conditional variances evolve over time following a specific process, then there will be periods of time when they are over the marginal (constant) variance and observations corresponding to these periods could be identified as outliers. Finally, if conditional heteroscedasticity and outliers appear together in a time series, genuine conditional heteroscedasticity could be masked by outliers, because they can distort the shape of the correlogram of squared residuals.

To illustrate the effects of the presence of outliers on residual diagnostics designed to detect conditional heteroscedasticity, in this section we generate several time series by ARIMA models with additive outliers. Also, we will simulate conditionally heteroscedastic series and test for outliers. Finally, we generate series with both outliers and conditional heteroscedasticity.

Conditional means of prices have been generated by stationary seasonal multiplicative ARIMA models. In particular, we choose the ARIMA (0,2,1) x $(1,0,0)_{12}$ given by:

$$(1 - \phi_{12} L^{12}) \Delta^2 p_t = (1 - \theta_1 L) a_t \qquad (1)$$

where p_t are nominal prices and a_t is a Gaussian white noise process. Model (1) has been chosen because it represents most of the empirical properties often found in the conditional mean of inflation; see, for example, Lorenzo (1997). Inflation is characterized by having a stochastic level with low persistence, reflected by the MA(1) parameter being close to unity, i.e. normal innovations have little effect over long-run expectations of inflation (Campbell and Mankiw, 1987). Model (1) also takes into account the seasonal correlations usually detected in monthly series of inflation. In particular, to represent these properties, we generate series with $\phi_{12} = 0.5$ and $\theta_1 = 0.9$. Most of the results presented in this section are robust to the model chosen for the conditional mean. We will point out when this is not the case. All the series have been generated using the SCA program of Liu and Hudak (1992) with a sample size of 340 observations and $a_t \sim NID(0, 1)$. Then, we through out the first 100 observations and work with the other 240 observations.

To analyse the effect of the presence of outliers over tests for conditional heteroscedasticity, we generate time series with both additive and level shift outliers using the following model:

$$\Delta^2 p_t = \omega_1 \Delta^2 D^1{}_t + \omega_2 \Delta^2 D^2{}_t + (1 - 0.9\, L) / (1 - 0.5\, L^{12}) \, a_t, \qquad (2)$$

where $D^1{}_t$ and $D^2{}_t$ are dummy variables. For additive outliers, these variables are pulse variables taking value 1 at time t = 40 and t = 160 respectively. For level shifts, the variables are step changes taking value 1 after t = 40 and t = 160 respectively. The parameters, ω_1 and ω_2, have been chosen equal to 3, 4.5 and 6 times the marginal standard deviation of the innovation a_t. After generating 100 time series, we estimate[4] the "true" ARIMA$(0,2,1)\times(1,0,0)_{12}$ model for each of them and compute some sample moments of the residuals, in particular, autocorrelations of squared residuals and their coefficient of kurtosis.

In table 1.A, we report results of the mean values of the residual moments. In table 1.B, we report, as an example, the results for one of the simulated series. As expected, the presence of outliers produces excess kurtosis both for additive and level shift outliers. However, the results on the autocorrelations of squares are different depending on the class of the outlier. For additive outliers, there is a strong first order autocorrelation and for level shifts there are not significant autocorrelations. This difference could be due to the fact that after taking two differences the residuals corresponding to an additive outlier show two extreme consecutive values of opposite sign while, if the outlier is a level shift, there is only one extreme value. However, notice that the effect in the case of the step variable depends on the value of the moving average parameter. In model (1), this parameter equals 0.9 being near to cancel the unit root in the autoregressive part of the model. If this parameter were smaller, implying more persistence in levels, then the effect of a step variable should be similar to the effect of a pulse, generating a strong first order autocorrelation in the squares. Finally, note that the outliers detected in the series used as an example, are exactly the outliers present in the data.

[4] All estimations of ARIMA models have been carried out using the exact estimation procedure of the SCA program. The identification of outliers have also been carried out with the SCA program.

Table 1

Sample moments of residuals of ARIMA(0,2,1)x(1,0,0)$_{12}$ models

A. Time series simulated with normal innovations, $a_t \sim N(0,1)$
Outliers at t=40 and t=160.
Number of simulations: 100

	pulse variables			**step variables**		
	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$
Mean	-0.0022	-0.0017	-0.0014	-0.0018	-0.0024	-0.0021
Standard	1.0787	1.1884	-1.3197	1.0273	1.0862	1.1613
Skewness	-0.0382	0.0983	-0.1627	0.1882	0.5460	1.0854
Exc. Kurtosis	1.0250	3.7326	6.1655	0.4911	2.3151	5.2071
r (1)	0.1730	0.3302	0.4048	-0.0078	-0.0062	-0.0050
r (2)	-0.0143	-0.0173	-0.0185	-0.0069	-0.0059	-0.0053
r (3)	-0.0063	-0.0163	-0.0190	0.0017	-0.0032	-0.0054
r (4)	-0.0015	-0.0105	-0.0149	-0.0038	-0.0045	-0.0051

B. Example

	pulse variables			**step variables**		
	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$
Mean	0.0993	0.1198	0.1406	0.0842	0.0927	0.1027
Standard eviation	1.0975	1.2133	1.3499	1.0280	1.0887	1.1388
Skewness	0.0011	-0.0293	-0.0860	0.1081	0.4989*	1.0906*
Exc. Kurtosis	1.0878*	3.6381*	7.4995*	0.4822	2.3887*	6.0533*
$r_2(1)$	0.35*	0.44*	0.47*	0.09	0.03	0.01
$r_2(2)$	-0.02	-0.03	-0.03	0.03	0.01	-0.01
$r_2(3)$	-0.06	-0.06	-0.05	-0.02	-0.05	-0.04
$r_2(4)$	0.02	0.01	-0.00	0.06	0.02	0.00
$Q_2(26)$	70.4*	78.1*	69.9*	27.3	19.1	11.8
Outliers **	40(4.28)A 160(6.02) A	40(6.34)A 160(7.35) A	40(8.08)A 160(8.31) A	40(3.27)I 160(4.2)A	40(4.75)I 160(5.11) A	40(6.11)I 160(5.88)I

* Significant at 5% level.
** "I" indicates that the outlier is innovative and "A" that it is additive.

The second set of simulated series is based on model (1) with the innovations, a_t, being substituted by the following process with conditional heteroscedasticity:

$$\varepsilon_t = \sigma^* a_t \sigma_t \qquad (3.a)$$
$$\log \sigma^2_t = \beta \log \sigma^2_{t-1} + \eta_t, \qquad (3.b)$$

where the constant σ^* is a scale parameter, $|\beta| < 1$ and η_t is a Gaussian white noise uncorrelated with ε_t. Model (3) is a stochastic volatility, ARV(1), model previously used in the literature to represent conditional heteroscedasticity in financial time series. The ARV model has some advantages over more traditional ARCH models to represent evolving variances; see, for example, Harvey *et al.* (1994). The disadvantage of stochastic volatility models with respect to models of the GARCH class is that their estimation by maximum likelihood based methods can only be carried out using computer intensive techniques. However, a quasi-maximum likelihood (QML) method is relatively easy to apply and is often reasonably efficient; see Harvey *et al.* (1994). This is the estimation approach taken in this paper.

Once more, we generate series by model (1) with ε_t instead of a_t. We are considering three parameter sets for the ARV model: *i)* $\beta = 0.95$ and $\sigma^2_\eta = 0.1$, *ii)* $\beta = 0.98$ and $\sigma^2_\eta = 0.05$, and *iii)* $\beta = 0.99$ and $\sigma^2_\eta = 0.03$. These values have been chosen because they are close to the values often estimated with real data. Then, we estimate the "true" ARIMA model and compute the residuals. In table 2.A we report, the mean value of the sample moments of the residuals for the 100 replicates and in part B, the results for one of the simulated series. As expected, in table 2 we can observe that conditional heteroscedasticity in the disturbances can cause excess kurtosis and significant autocorrelations of squared residuals. In contrast with the results reported in table 1, these autocorrelations are not only significant for the first lag. Notice that, although the series considered in table 2 have been generated without outliers, the usual procedures detect outliers who usually are innovative and sometimes bigger than 4 standard (marginal) deviations. Therefore, it is necessary to be very cautious before removing outliers from heteroscedastic time series.

In sum, comparing tables 1 and 2, we can observe that both outliers and conditional heteroscedasticity, generate excess kurtosis and significant autocorrelations of squared residuals. Therefore, both effects can be confused when looking at usual diagnostic tests. Furthermore, conditional heteroscedasticity can generate what can be identified as outliers by traditional procedures.

Finally, we simulate series by model (1) with both conditional heteroscedasticity and outliers. Then, we estimate model (1) for each of the series and compute the mean of the sample moments of the residuals. The results appear in table 3, where we can observe that the presence of level shift outliers in a heteroscedastic time series can distort the correlogram of squared residuals with none of the autocorrelations being significant. On the other hand, when the outliers are additive, there is a strong first order autocorrelation with all other autocorrelations being not significant, i.e. a pattern similar to a MA(1) model. In table 2, we have seen that when the series are generated by an heteroscedastic model without outliers, the pattern of the autocorrelations of the squared residuals is similar to an ARMA(1,1) model. Therefore, genuine heteroscedastic effects can be masked by outliers. Finally, notice that among the outliers identified in table 3 are the genuine outliers, and that they are the biggest in terms of the standard deviation. However, looking at the outliers detected in table 3, it is difficult to decide which are genuine and which are effect of the conditional heteroscedasticity.

Table 2

Sample moments of residuals of ARIMA(0,2,1)x(1,0,0)$_{12}$ models

A. Time series simulated with ARV(1) innovations
Number of simulations: 100

	$\phi = 0.95, \sigma_\eta^2 = 0.1$	$\phi = 0.98, \sigma_\eta^2 = 0.05$	$\phi = 0.99, \sigma_\eta^2 = 0.03$
Mean	0.0016	0.0005	-0.0019
Standard Deviation	0.9442	0.9158	0.8942
Skewness	0.0830	0.0566	0.0366
Exc. Kurtosis	2.4755	2.0612	1.5518
$r_2(1)$	0.1447	0.1316	0.1141
$r_2(2)$	0.1471	0.1477	0.1354
$r_2(3)$	0.1429	0.1445	0.1327
$r_2(4)$	0.1208	0.1310	0.1191
$3\ \sigma_\varepsilon \leq$ outliers $< 3.5\ \sigma_\varepsilon$	4.17	3.65	3.59
$3.5\ \sigma_\varepsilon \leq$ outliers $< 4\ \sigma_\varepsilon$	1.49	1.75	1.89
outliers $> 4\ \sigma_\varepsilon$	1.51	1.23	0.82
Total Outliers	7.17	6.63	6.30

B. Example

	$\phi = 0.95, \sigma_\eta^2 = 0.1$	$\phi = 0.98, \sigma_\eta^2 = 0.05$	$\phi = 0.99, \sigma_\eta^2 = 0.03$
Mean	0.0643	0.1125	0.0743
Standard Deviation	0.8756	0.9417	0.9142
Skewness	-0.2769	-0.1365	-0.0667
Exc. Kurtosis	1.6497*	1.2166*	0.7258*
$r_2(1)$	0.24*	0.18*	0.22*
$r_2(2)$	0.09	0.04	0.04
$r_2(3)$	0.24*	0.22*	0.15*
$r_2(4)$	0.35*	0.26*	0.25*
$Q_2(26)$	81.4*	51.8*	48.5*
Outliers **	107 (-4.50) I 111 (3.62) I 104 (-3.26) I 146 (3.10) A	107 (-4.11) I 111 (3.47) I 146 (3.17) A 104 (-3.23) I	107 (-3.74) I 111 (3.21) I 146 (3.24) A 104 (-3.06) I 173 (3.01) I 174 (3.05) I

* Significant at 5% level.

** “I” indicates that the outlier is innovative and “A” that it is additive.

Table 3

Sample moments of residuals of ARIMA(0,2,1)x(1,0,0)$_{12}$ models
Time series simulated with ARV(1) innovations
Outliers at t=40 and t=160
Number of simulations: 100

$\phi = 0.95$, $\sigma_\eta^2 = 0.1$

	pulse variables			step variables		
	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$
Mean	0.0021	0.0058	0.0040	0.0027	0.0034	0.0041
Standard Deviation	1.0502	1.1631	1.2980	0.9966	1.0595	1.1377
Skewness	-0.0034	-0.0815	-0.1526	0.3259	0.7570	1.3759
Exc. Kurtosis	2.9757	4.9467	3.7817	2.6353	4.3744	4.3874
$r_2(1)$	0.2378	0.3391	0.3991	0.1032	0.0624	0.0340
$r_2(2)$	0.0710	0.0254	0.0035	0.1012	0.0572	0.0294
$r_2(3)$	0.0656	0.0218	0.0008	0.0952	0.0546	0.0274
$r_2(4)$	0.0593	0.0205	0.0020	0.0785	0.0443	0.0216

$\phi = 0.98$, $\sigma_\eta^2 = 0.05$

	pulse variables			step variables		
	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$
Mean	0.0020	0.0064	0.0075	0.0012	0.0013	0.0015
Standard Deviation	1.0302	1.1466	1.2849	0.9719	1.0377	1.1195
Skewness	-0.0403	-0.1265	-0.1879	0.4212	0.9956	1.7264
Exc. Kurtosis	3.5757	3.9073	3.2938	2.9916	3.5939	3.5724
$r_2(1)$	0.2593	0.3494	0.4019	0.0806	0.0472	0.0260
$r_2(2)$	0.0582	0.0209	0.0024	0.0885	0.0500	0.0268
$r_2(3)$	0.0558	0.0191	0.0017	0.0841	0.0471	0.0245
$r_2(4)$	0.0537	0.0201	0.0041	0.0725	0.0432	0.0242

$\phi = 0.99$, $\sigma_\eta^2 = 0.03$

	pulse variables			step variables		
	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$	$3\sigma_a$	$4.5\sigma_a$	$6\sigma_a$
Mean	-0.0004	0.0040	0.0055	-0.0012	-0.0009	-0.0007
Standard Deviation	1.0204	1.1432	1.2849	0.9594	1.0301	1.1157
Skewness	-0.0848	-0.1678	-0.2153	0.6521	1.4085	2.2417
Exc. Kurtosis	3.1043	3.2771	2.4822	2.5985	2.9981	3.1925
$r_2(1)$	0.2814	0.3631	0.4074	0.0606	0.0364	0.0216
$r_2(2)$	0.0435	0.0147	0.0005	0.0710	0.0402	0.0222
$r_2(3)$	0.0402	0.0115	-0.0007	0.0661	0.0357	0.0189
$r_2(4)$	0.0405	0.0155	0.0031	0.0584	0.0367	0.0225

In this section, we have illustrated that the effects of conditional heteroscedasticity and outliers can be badly confused. As the economic interpretation of both phenomena is rather different, it is important to distinguish between them when analyzing real data. In particular, when analyzing the relation between the level and uncertainty of inflation, the implications of both phenomena are different in terms of the hypothesis to be tested. However, it is difficult to see how to solve this methodological problem. At the moment, available methods for detection of outliers are based on assuming that the variance is constant over time. As previously illustrated, these methods seem to be not adequate under conditional heteroscedasticity. On the other hand, as proposed by van Dijk et al. (1996), we can use robust methods to test and estimate the conditional heteroscedasticity model. However, it seems to us that these methods are once more based on identifying observations which are abnormal with respect to a constant standard deviation and, consequently, using them we can loose information on the "true" heteroscedastic model. That is the reason why, to analyse the inflation series in the next section we are first, considering ARIMA models with intervention analysis only for the very “big” outliers, say bigger than 4.5 marginal standard deviations. Then, we fit a model for the variance and use the observations standardized using the conditional standard deviation, to identify further outliers. If "conditional" outliers are present, we include new interventions in the model and start the procedure until the residuals look as a Gaussian white noise.

3. EMPIRICAL ANALYSIS OF INFLATION

3.1 Models for conditional means

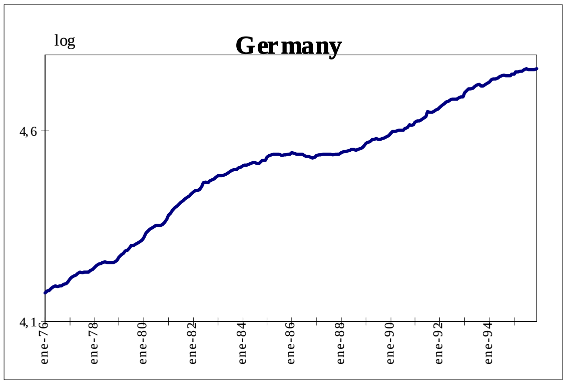
In this section we are modeling monthly observations of inflation measured by the first difference of logarithmic prices from January 1976 to December 1995 for the G7 countries. The series have been plotted in figure 1. Each inflation series has been transformed by taking first differences to achieve stationarity in the mean.

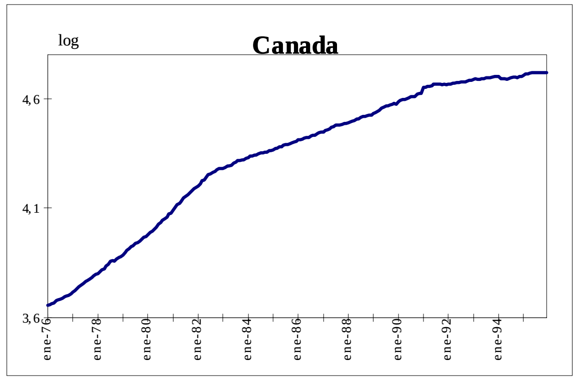
All inflation series have been modeled by seasonal ARIMA(p,2,q)$\times$(P,0,Q)$_{12}$ models[5]. Some sample moments of the residuals from each of the countries appear in table 4 where it is possible to observe that, with the exception of Japan, all series of residuals have excess kurtosis and all except Japan and US have asymmetric distributions. With respect to the autocorrelations of squared residuals, Germany, U.S. and U.K. have a strong first order autocorrelation, Canada, France and Japan have no significant autocorrelations and Italy have a strong significant autocorrelation at lag four. In all the series several outliers are identified.

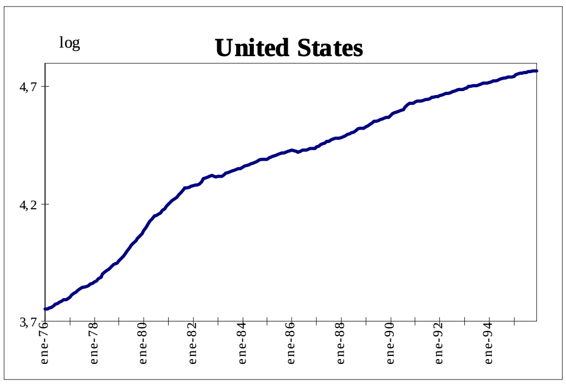
[5] Details on stationarity transformations and estimated models available from the authors upon request.

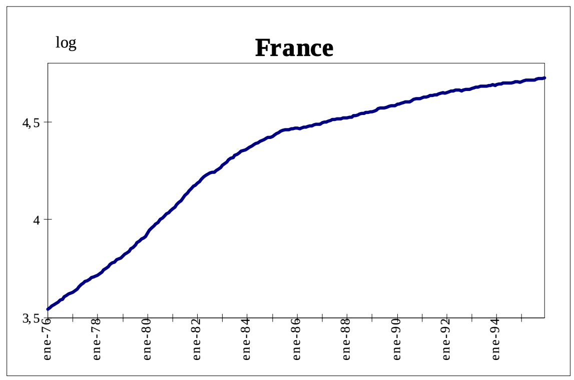
Figure 1

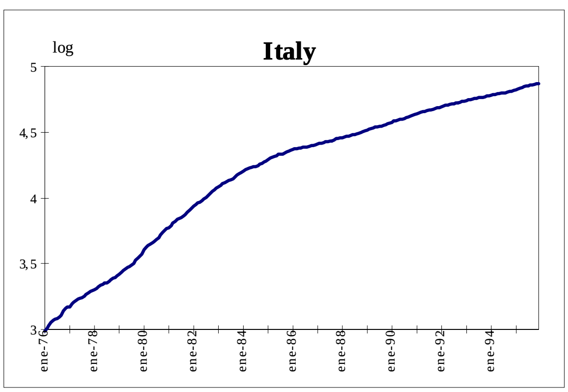
Monthly CPI data for G-7 countries
Period: 1976.01 - 1995.12
(logarithms)

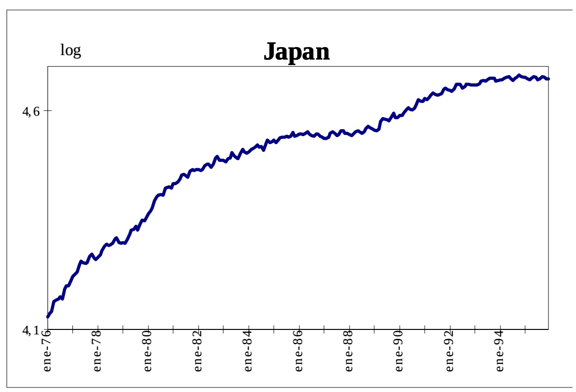


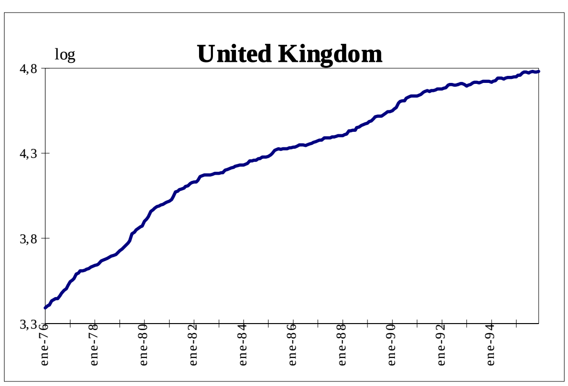












We consider the intervention of some of the "big" outliers identified in table 4. We choose as big, residuals over 3.75 times the standard deviation of the innovations. Consequently, we re-estimate the multiplicative ARIMA models with dummy variables for such interventions. The sample moments of the residuals from these models are reported in table 5 where we can observe that, after modeling the interventions, there is a clearer autocorrelation pattern in the squared residuals of Canada, US, France and Italy suggesting the presence of conditional heteroscedasticity. Germany and UK seem to have no conditional heteroscedasticity, meaning that inflation uncertainty has a constant mean. Finally, Japan has only a strong first order autocorrelation. There are still several outliers identified in table 5 but none of them is "big" enough as to be considered for intervention at the moment. Comparing tables 4 and 5 we can also notice that the Skewness has been clearly reduced, being not significant for any of the series analyzed. Also, the excess kurtosis parameter is not significant, except for UK inflation.

Table 4

Sample moments of residuals of ARIMA models without Intervention Analysis
Estimation sample: 1976.01 - 1995.12

	Germany	**Canada**	**United States**	**France**	**Italy**	**Japan**	**United Kingdom**
St.	0.0023	0.0032	0.0024	0.0021	0.0032	0.0038	0.0045
Skewness	0.64*	0.81*	-0.42	0.27	0.72*	0.27	2.36*
Ex.	2.69*	6.06*	2.17*	3.40*	6.19*	0.02	18.72*
$r_2(1)$	0.19*	0.05	0.27*	0.00	0.07	0.07	0.18*
$r_2(2)$	-0.02	0.08	0.05	0.03	0.08	-0.02	0.00
$r_2(3)$	0.01	0.12*	0.07	0.01	0.04	0.02	-0.01
$r_2(4)$	-0.04	-0.01	0.05	0.14*	0.37*	-0.06	-0.03
$Q_2(26)$	24.7	12.1	90.2	27.4	146.0*	26.0	16.6
Outliers	43(3.10)A 50(3.48)I **178(4.35)A·** **187(5.53)A·**	33(-3.93)I 85(-3.21)I **181(6.72)I·** 218(-3.29)I	**55(-5.07)I·** 83(-3.22)I 88(3.07)I 169(3.37)I	**49(4.80)I·** **79(-4.85)I·** **133(3.98)I·**	24(-3.24)A 26(-3.04)A 37(3.02)I **45(5.32)I·** **49(5.70)I·** 54(-3.19)A **61(-5.42)A·** 80(3.03)I	43(3.29)A 104(-3.06)A **160(3.82)I·**	**43(9.69)I·** **44(-3.92)I·** **48(-3.77)A·** 104(3.13)I **172(3.81)I·** 179(-3.11)I 205(-3.15)A

* Significant at 5% level.
** "I" indicates that the outlier is innovative and "A" that it is additive.
· "Big" outliers.

Table 5

Sample moments of residuals of ARIMA models with Intervention of "big" outliers
Estimation sample: 1976.01 - 1995.12

	Germany	Canada	United States	France	Italy	Japan	United Kingdom
St.	0.0020	0.0028	0.0023	0.0018	0.0026	0.0035	0.0033
Skewness	0.2448	-0.2862	-0.0135	-0.1876	0.0598	0.1002	0.1718
Ex.	0.3711	0.6088	0.3807	0.4643	0.4888	-0.6277	0.8476*
$r_2(1)$	-0.09	0.15*	0.12*	0.16*	0.08	0.15*	0.07
$r_2(2)$	0.03	0.12*	0.13*	0.10	0.13*	0.04	0.07
$r_2(3)$	0.17*	0.05	0.13*	0.05	0.07	0.08	-0.01
$r_2(4)$	-0.08	0.14*	0.14*	-0.03	0.21*	0.04	0.02
$Q_2(26)$	37.6	24.3	75.8*	31.4	165.0*	31.2	23.1
Outliers	43(3.35)A 50(3.43)I	33(-4.02)I 85(-3.33)I 218(-3.43)I	69(3.21)A 169(3.32)I	37(3.12)A 40(-3.03)A 80(-3.30)I 81(-3.14)I 82(-3.04)A 122(-3.11)I 235(-3.01)A	26(-3.10)A 67(-3.54)I 80(3.12)I	43(3.41)A	52(3.81)I 104(3.10)I 179(-3.12)I 205(-3.03)A

* Significant at 5% level.
** "I" indicates that the outlier is innovative and "A" that it is additive.

3.2 Models for conditional variances

In this section we are fitting the ARV(1) model in (3) to the residuals from the ARIMA models with intervention analysis. As mentioned previously, to estimate the parameters of the volatility process, we use a QML method as described in Harvey *et al.* (1994) applied to the standardized residuals. Then we use the Kalman filter and a smoothing algorithm to obtain estimates of the volatility, $\hat{\sigma}_{t/T}$. The scale parameter, σ^*, can be estimated by the sample variance of the heteroscedasticity corrected residuals; see Harvey and Shephard (1993). In table 6 we report the estimated values of the volatility parameters together with some sample moments of the standardized residuals, i.e. $\hat{a}_t = \hat{\varepsilon}_t / \hat{\sigma}^* \hat{\sigma}_{t/T}$, which should behave as a standard normal white noise process. We can observe that the estimates of the parameter β are close to unity for Italy, Japan and UK, suggesting persistence of the conditional variance. For these three countries, the squared standardized residuals have not any more significant autocorrelations. Furthermore, there are not outliers with respect to the conditional deviation, meaning that the outliers detected in table 5 could be due to the conditional heteroscedasticity. However, we detect "conditional outliers" for Germany, Canada, France, and US. These outliers were also detected in table 5. However, in table 5, we identified outliers which do not appear in table 6. Finally, we incorporate the "conditional outliers" identified in table 6 to the ARIMA models and reestimate the models.

Table 6

Estimated ARV(1) models after intervention of "big" outliers

	Germany	**Canada**	**United States**	**France**	**Italy**	**Japan**	**United Kingdom**
σ	$3.34 \times 10^{-}$	$5.99 \times 10^{-}$	4.72×10^{-6}	$2.79 \times$	$5.60 \times$	$1.19 \times$	9.34×10^{-6}
ϕ	0.7040	0.9143	0.5733	0.8926	0.9850	0.9846	0.9190
σ_η^2	0.3301	0.0577	0.1095	0.0872	0.0197	0.0073	0.0726
σ_ξ^2	4.9348**	4.0642	5.9672	4.4222	5.2375	3.7476	3.5658
logL	-277.97	-61.43	-315.05	-270.06	-304.48	-263.64	-264.14
Skewness	0.1138	-0.0180	0.0270	0.0109	0.1166	0.0389	0.2594
Ex.	0.0335	-0.2926	0.2633	0.1649	-0.1233	-0.5347	0.4783
$r_2(1)$	-0.0836	-0.0501	0.0697	0.0128	-0.0796	0.0517	-0.0374
$r_2(2)$	-0.1401	-0.0189	0.0868	-0.0515	-0.0464	-0.0215	0.0448
$r_2(3)$	0.0372	0.0410	0.1081	-0.0468	-0.0240	0.0164	-0.1025
$r_2(4)$	-0.1298	0.1454	0.0999	-0.0721	-0.0147	-0.0584	-0.0318
$Q_2(10)$	19.88	10.71	38.95*	5.53	6.10	9.28	11.26
Conditional Outliers	50 (3.40)	---	169 (3.27)	236(3.24)	---	---	---
** Parameter fixed in the estimation process.							

Table 7 shows the estimated parameters of the ARV(1) model and sample moments of the standardized residuals from the ARIMA models with "conditional" interventions. After introducing the "conditional" interventions in the ARIMA models, the estimates of the parameter β change fundamentally for Canada and US, being much closer to unity than in table 6. However, the estimates for France do not change much suggesting constant uncertainty. Finally, it is important to note that the estimates for Germany, although have changed, keep indicating a constant conditional variance. In table 7, we also observe that there are not any significant autocorrelations in the squared standardized residuals. Therefore, the stochastic volatility model has been able to properly represent the dynamic behavior of the uncertainty of the inflation series. Only, the volatility of inflation in Germany seems to have specification problems. In this case, to obtain sensible estimates we are forced to keep the parameter σ^2_ξ = 4.9348 during the estimation procedure. This value of the parameter is the one implied if the distribution of ε_t were conditionally normal. Notice, that the estimates of σ^2_ξ for all other countries do not suggest important deviations from the normality hypothesis. Finally, we could not find any more "conditional outliers" for any of the inflation series. Figures 2 to 4 represent the smoothed estimates of the uncertainty of inflation for each of the countries analyzed. Comparing the plots in these figures, we may observe that the uncertainty of inflation in USA, Canada and UK have very similar shapes, with a big increase in the early 80´s going down afterwards due to the control of inflation policies. On the other hand, Italy and Japan may have a change in the level of the volatility process. Finally, the uncertainty in France seems to fluctuate around a constant level, while in Germany we could not reject the homoscedasticity hypothesis.

Table 7

Estimated ARV(1) models after intervention of "big" and conditional outliers

	Germany	United States	France
σ	3.00×10^{-6}	4.33×10^{-6}	2.63×10^{-6}
ϕ	0.9919	0.9506	0.8843
σ_η^2	0.0000	0.0166	0.0874
σ_ξ^2	4.9398	4.7817	4.1788
log L	-283.08	-290.44	-264.17
Skewness.	-0.077	-0.0320	-0.1839
Exc. Kurtosis	-0.341	-0.2404	-0.0407
$r_2(1)$	-0.079	0.0706	0.0035
$r_2(2)$	-0.103	0.0746	-0.0022
$r_2(3)$	0.101	0.0745	-0.0033
$r_2(4)$	-0.119	0.0786	-0.0435
$Q_2(10)$	17.77	11.16	3.58
Conditional Outliers	---	---	---

Summarizing, after taking into account the effects of outliers on the inflation series, USA, Canada and UK seem to have conditional heteroscedasticity with high persistence in the uncertainty process. France show a shape of volatility slightly changing around a constant level, and Germany exhibit a constant level of inflation uncertainty. Finally, Italy and Japan may have a change in the unconditional variance of inflation.

4. RELATION BETWEEN LEVEL AND UNCERTAINTY OF INFLATION

There are important methodological limitations to test the Friedman hypothesis. Ba` illie *et al*. (1996) carried out likelihood ratio tests on whether lagged inflation Granger causes volatility and of whether lagged volatility Granger causes inflation. However, as we have seen previously, estimated volatilities are close to be not stationary and in such circumstances, usual tests of causality may not be reliable. Furthermore, as suggested by Cosimano and Jansen (1988), the relation between the level and uncertainty of inflation may depend on the level and, consequently, is not constant over time. Finally, some authors suggest a relation between level and uncertainty only in the long run. In order to obtain a first approximation to the nature of the relationship between the level and uncertainty of inflation for the G-7 countries, figures 2 o 4 represent the smoothed estimates of volatility together with smoothed estimates of inflation levels for each of such countri[e]sˆ . When looking at the relation between uncertainty and the level of inflation, we may observe three different patterns. First, consider the cases of US, Canada and UK, represented in figure 2. For all three economies, it seems rather difficult to find a constant relationship between uncertainty and level of inflation for the whole sample period. Up to approximately mid 80´s, the reduction of inflation levels seems to be followed by a gradual reduction in inflation volatility, supportg the Friedman hypothesis. However, during the period of control of inflation

6 Inflation levels in figures 2, 3 and 4 have been obtained using the program STAMP 5.0 (Koopman *et al.*, 1995).

this relation seems to disappear. This may support the results of Cosimano and Jansen (1988) who find the relation only when inflation is high.

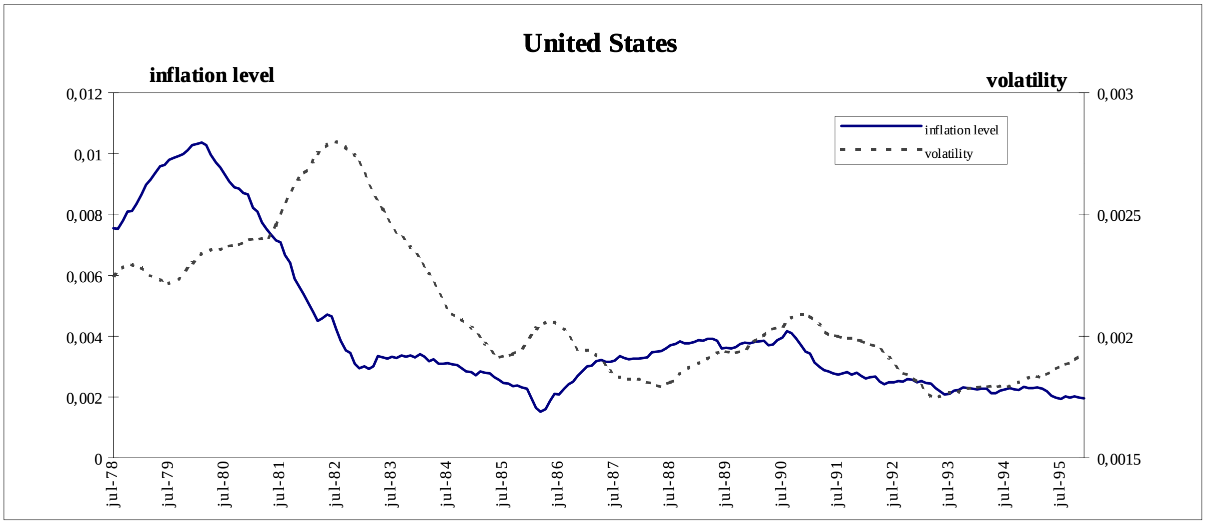
When looking at the plots of inflation and uncertainty corresponding to France and Germany in figure 3, we observe that the level of inflation is evolving over time, while the uncertainty fluctuates around a constant level. Consequently, we do not observe any relationship supporting the Friedman hypothesis for these countries.

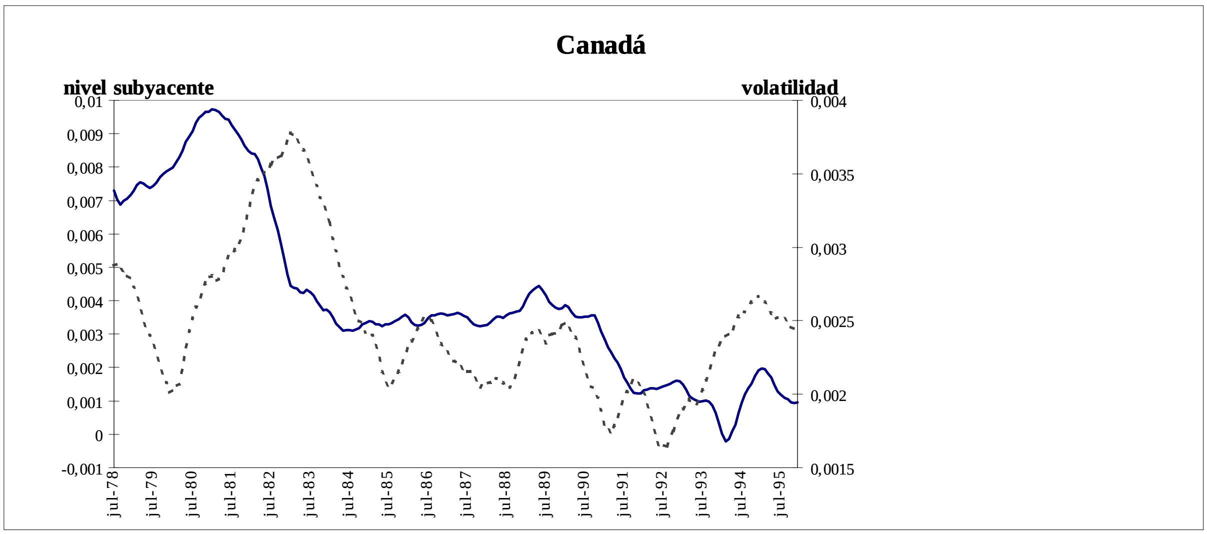
Finally, figure 4 represents inflation level and uncertainty for Italy and Japan. When looking at the relationship in Italy, both the level and the uncertainty of inflation seem to have very similar shapes over time, supporting the hypothesis that greater inflation rates are associated with greater uncertainty about future inflation. There is a structural change from high levels of inflation and uncertainty to lower ones with a very smooth adjustment to the new level. Finally, in Japan the deceleration of inflation is accompanied by a systematic reduction of uncertainty. However, the level of inflation has a rather quick adjustment to the new level, while the uncertainty adjustment is much slower. Furthermore, during 1986 and up to 1990 the level of inflation increased and then has an important decrease with no significant movements in uncertainty.

Summarizing the conclusions from previous figures, we can see that there are important heterogeneity's in the joint evolution of inflation levels and uncertainty in the different countries of the G-7 group. The empirical evidence presented suggests that the relationship between the level and uncertainty of inflation presents different characteristics depending on the country and perhaps, on the period considered. Such relationship seems far from being simple and homogeneous among the economies of the G-7 group. For an empirical analysis of such relationship, it seems necessary to use multivariate non-linear models able to represent the short and long run relationships between levels and volatility and this is beyond the scope of this paper.

Figure 2

Inflation level and volatility





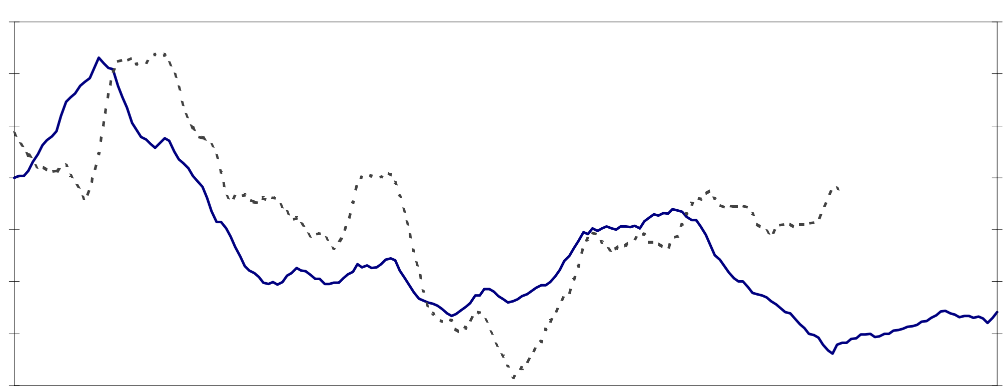
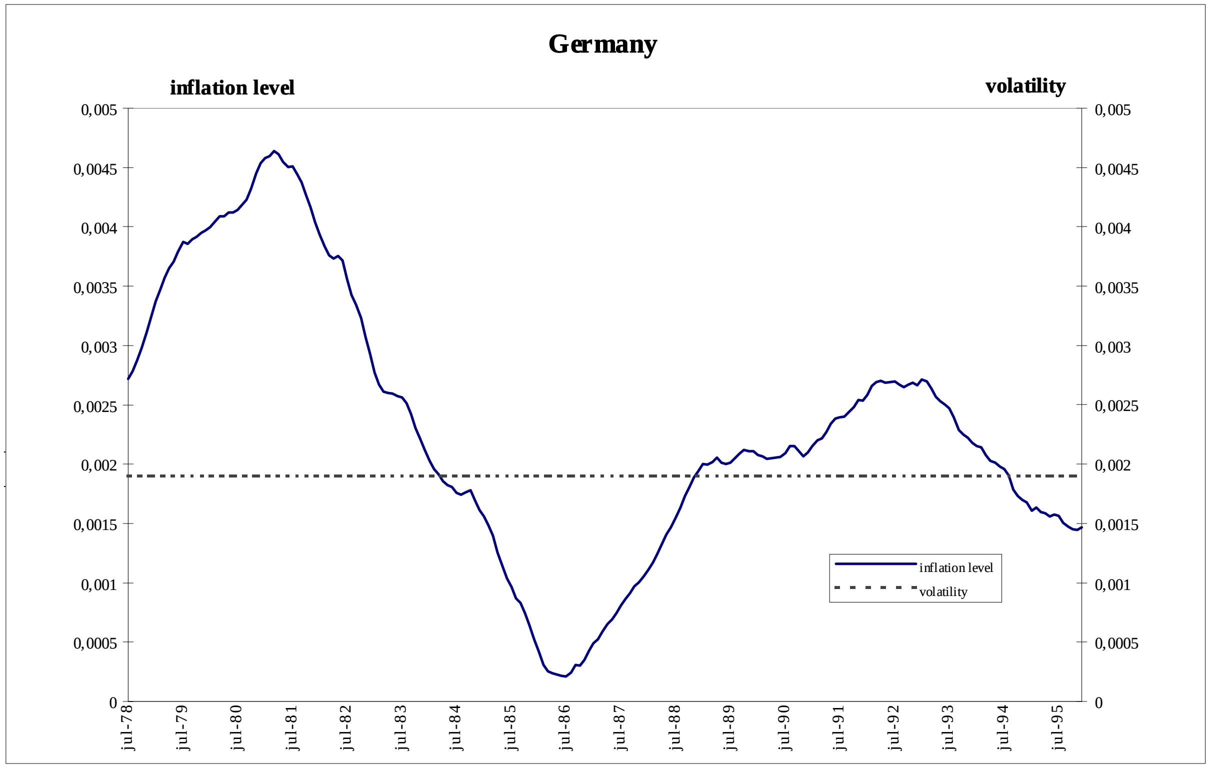


Figure 3

Inflation level and volatility



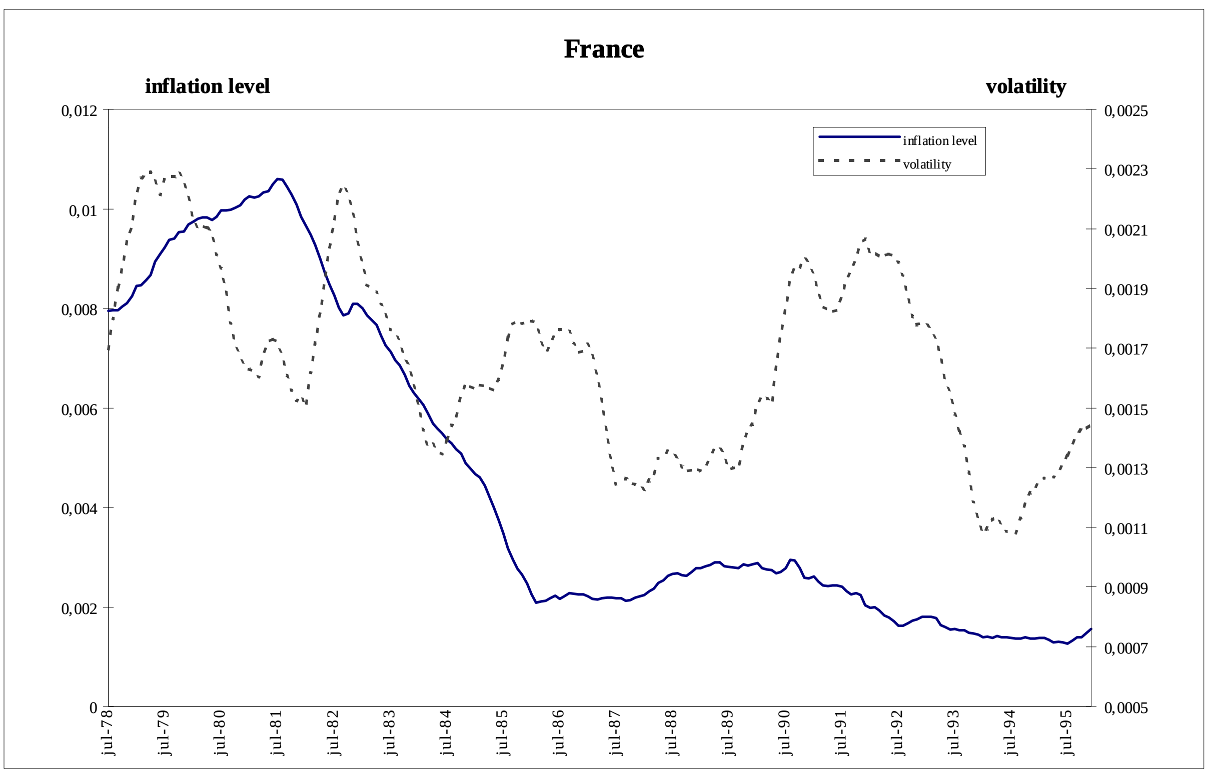
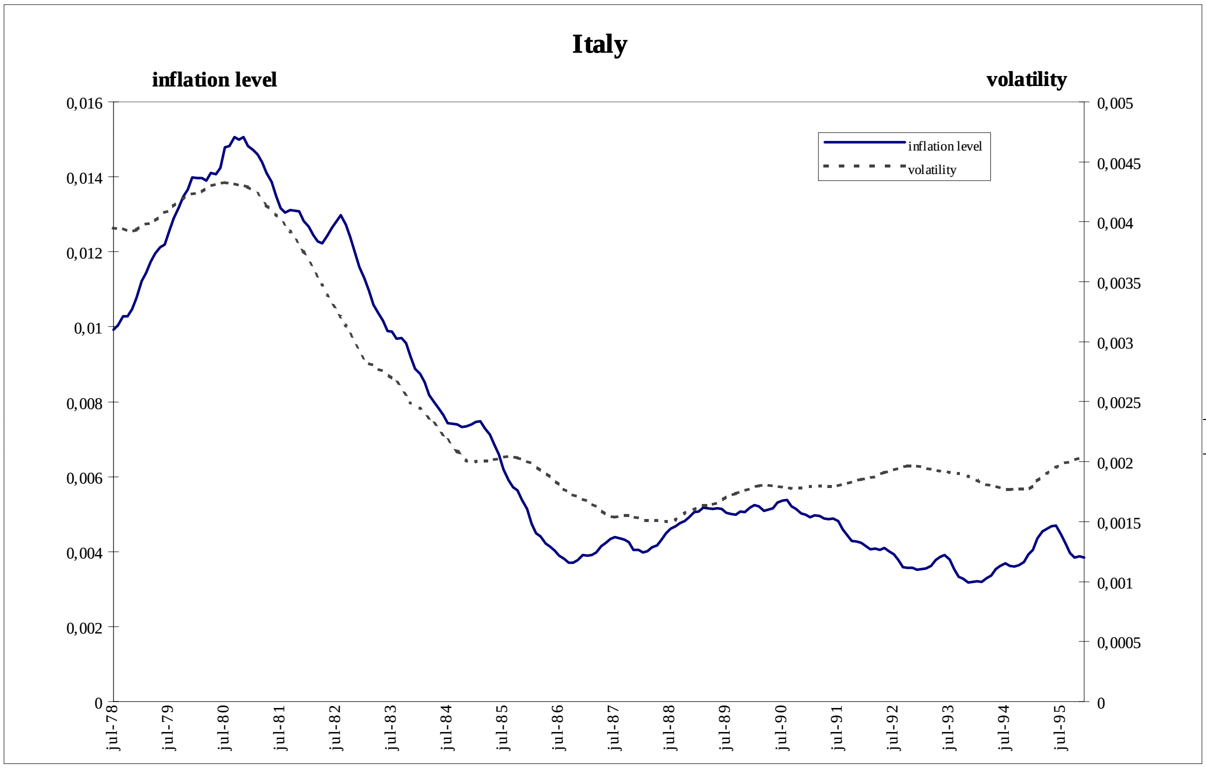
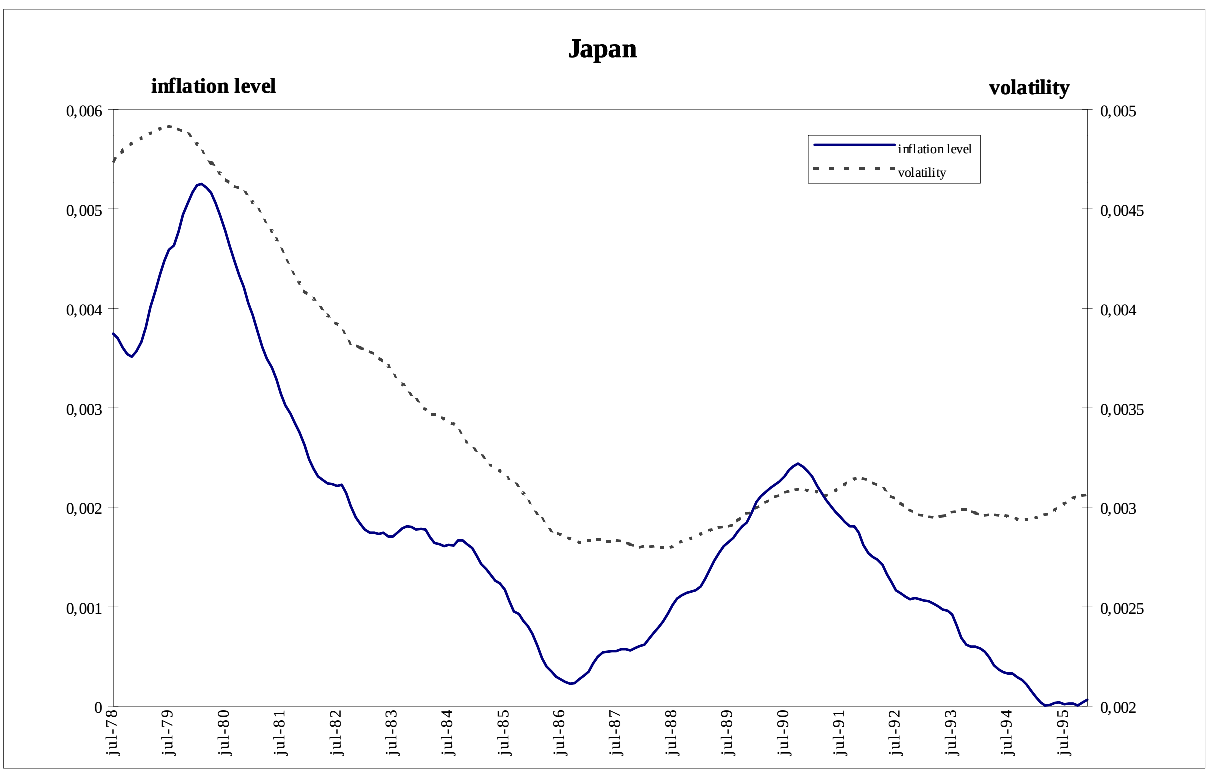


Figure 4

Inflation level and volatility





5. CONCLUSIONS AND FINAL REMARKS

The hypothesis of a positive relationship between the level and uncertainty about the level of inflation is often postulated by economists. It seems rather clear that univariate analysis of monthly series of inflation suggests that there is some kind of non-linearity which could be due to the presence of conditional heteroscedasticity. On the other hand, inflation series are often affected by the presence of outliers which may affect substantially the modeling of their dynamics. Since conditional heteroscedasticity and outliers have rather different economic interpretations, it is fundamental to properly separate them.

From an econometric point of view, we have illustrate with simulations that, when both effects appear together in a time series, we can identify as outliers observations corresponding to periods of time when the conditional variance is over the marginal variance and that usual diagnostics on the residuals have difficulty detecting genuine conditional heteroscedasticity.

In this paper we are not proposing a methodological solution to this problem. Our proposal is empirical and consists in modeling first interventions only for the very big outliers. Then, we estimate the stochastic variance model of the residuals which are standardized using the smoothed estimates of the conditional variance. The standardized residuals are then used to find further "conditional outliers" which are included as additional interventions in the model. Then, we re-estimate the intervention ARIMA model and fit the stochastic volatility model to the new residuals. Applying this strategy to inflation series for the G-7 group countries we find that heteroscedasticity effects which were not clear in the first step appear clearly after including interventions for the "big" outliers. The estimation of the heteroscedasticity model seems to be very sensible to the presence of outliers in the series.

With respect to the results relating the estimates of the uncertainty of inflation, we find that its evolution over time follows different patterns in the different countries. Finally, we also find that there are important heterogeneities in the joint evolution of inflation levels and uncertainty. The empirical evidence presented suggests that the relationship between the level and uncertainty of inflation presents different characteristics depending on the country and perhaps, on the period considered. Such relationship seems far from being simple and homogeneous among the economies of the G-7 group.

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