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Structural breaks and GARCH modeling.

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In this paper we propose a model which nests an endogenously determined structural shift in variance with the standard GARCH model. The structural shift element of our approach is based on the Hamilton(1988, 1991) endogenous switching markov process model and we show how this may be combined with a GARCH variance process to nest the two models. A natural testing procedure then arises to select between the two alternatives, or, of course, to remain with the more general model.

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1 INTRODUCTION

Bollerslev and Engle (1986) introduced the notion of integrated in variance generalized autoregressive conditional heteroscedastic (IGARCH) models. These models possess a unit root in the variance process and so they characterise a condition of persistence in variance. Since the inception of such a model, empirical work has found a surprisingly large number of cases of such IGARCH behaviour. There are a number of explanations for this finding. It is conceivable that the variance of a process is related to some nonstationary variable; in this case the IGARCH model is a misspecified approximation to the true variance process. Another explanation, following Nelson (1990) and Drost and Nijman (1990) is based on temporal aggregation and suggests that a low data frequency GARCH(1,1) process may be well approximated at high frequencies by an IGARCH process. The third explanation may be traced back to Diebold (1986) and parallels Perrons' (1989) analysis of integration in mean; if structural breaks take place in the variance process which are not allowed for in the model, then an upward bias will result in the parameter estimates leading to the appearance of an IGARCH process. Lamoreux and Lastrapes (1990) present monte carlo evidence which demonstrates this point. In this paper we wish to pursue this final point further by proposing a model which nests an endogenously determined structural shift in variance with the standard GARCH model. The structural shift element of our approach is based on the Hamilton (1988, 1991) endogenous switching markov process model and we show how this may be combined with a GARCH variance process to nest the two models. A natural testing procedure then arises to select between the two alternatives, or, of course, to remain with the more general model.

We illustrate the estimation strategy with two data sets, both of which appear to exhibit strong GARCH process on standard procedures; the spread on three to six month US treasury bills and the three month US treasury bill rate itself. The more general model finds clear evidence for one of these series being best described by a standard GARCH process while the other is clearly characterised by a discrete change in variance.

The plan of the paper is as follows; in section 2 we outline the Hamilton filter and some generalisations of the switching model implemented by Hamilton and show how the standard GARCH formulation may be put into this framework; section 3 then discusses the problem of testing from this general model; section 4 then examines our two data sets; section 5 presents our conclusions.

2 SWITCHING AND GARCH

In this section, we first outline the basic Hamilton filter in its most general form. We then go on to show how it may be generalized to allow for a switching autoregressive variance process. Related papers have investigated Markov switching ARCH models, notably Brunner (1991), Cai (1992) and Hamilton and Susmel (1992), but these papers have not considered the GARCH extension and have allowed only very limited forms of switching. Hamilton and Susmel, for example, impose unchanging weights on the ARCH process and simply allow an overall switching scaling factor for the variance in different regimes. Thus, it would be impossible to find one state with ARCH effects and another without.

The Hamilton (1990) Filter is a non-linear filter which allows the state equation to follow a particular non-linear restriction where the state variables follow a markov chain subject to a discrete adding-up restriction.

The general problem statement consists of the usual two parts; a set of measurement equations and a set of state equations.

The measurement equation may be written as

$$y_t = x_t' B \xi_t + \omega_t$$

$$\textit{where } \omega \sim NIID(0, \sigma^2 \gamma \xi_t),\ \xi_t \textit{ are the n state variables} \tag{1}$$

$$\textit{and } x_t' B \textit{ and } \gamma \textit{ are known}$$

the state equations take the usual form

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{2}$$

where F is an nxn matrix. Now if v_t were normally distributed this would be a standard state space model and the Kalman Filter would give optimal inference. The idea behind this model is, however, that each of the state variables represent the probability of being in a different state of the world (S_i, $i=1...n$). This implies that the state variables must sum to unity and, therefore, cannot be subject to the usual error process. The Hamilton Filter gives optimal inference and evaluates the likelihood function for this non-linear system. The motivation for this model is very strong as this system may be seen as a general regime switching framework. For example, if there are two state variables, this model encompasses the endogenous regime switching disequilibrium model.

The filter rests on the following statements, under the normality assumption made in the measurement equation,

$$f(y_t|S_t=i,X_t) = \frac{1}{(2\pi(\sigma S_t)^2)^{1/2}} \exp\left(\frac{-(y_t-x_t BS_t)^2}{2(\sigma S_t)^2}\right) \tag{3}$$

and of course

$$f(y_t,S_t=i|x_t) = f(y_t|S_t=i,x_t).prob(S_t=i|x_t) \tag{4}$$

and

$$f(y_t|x_t) = \sum_{i=1}^{n} f(y_t,S_t=i|x_t) \tag{5}$$

The optimal inference about S_t may then be based on

$$\xi_{ti} = Prob(S_t=i|x_t,y_t) = \frac{f(y_t,S_t=i|x_t)}{f(y_t|x_t)} \tag{6}$$

The one step ahead prediction errors may then be derived by using the state equation to produce a forecast of the state variables in t+1 and the measurement equation may be evaluated in the usual way. The likelihood function may then be evaluated.

This filter is in theory completely general as it allows for many states and all the parameters of the model are allowed to switch. In his applications, Hamilton (1988, 1989, 1990) implemented a very limited form of switching where only the constant and the variance of an equation were allowed to switch between regimes. Hall and Sola (1993) discussed various generalizations of this which allowed the full parameter vector to switch and they also discussed the implications of various possible assumptions about the treatment of dynamic effects.

Hall (1991) and Goodhart, Hall, Henry and Pesaran (1993) discuss formulating the GARCH model within a state space framework and estimating it using the Kalman filter. Hall (1991) takes this formulation and extends the standard GARCH-M model to include a stochastic component in the variance equation. The important point from our perspective is that once the GARCH model is formulated in state space it becomes relatively easy to merge it with the Hamilton filter. This can be done by restating (1) and (2) in the following more general form.

The general measurement equation may be written as

$$y_t = x_t'B\xi_t + \Gamma\xi_t\sigma^2_{t|t-1} + \omega_t$$

where $\omega \sim NIID(0,\sigma^2_{t|t-1})$, ξ_t *are the n state variables*

and $x_t'B$ *and the vectors* γ_i *are known and* **(7)**

$$\sigma^2_{t|t-1} = \gamma_0\xi_t + \gamma_1\xi_t\omega_{t-1} + \gamma_2\xi_t\sigma_{t-1|t-2}$$

and the state equations remain

$$\xi_{t+1} = F\xi_t + v_{t+1} \qquad (8)$$

This very general form allows switching in the parameters generating both the mean and the variance. Under the condition that $\Gamma \neq 0$ this model represents a switching GARCH in mean model (GARCH-M), we make this statement for generality but will not consider the GARCH-M extension in the examples given in this paper. If the parameters of the model do not differ significantly, then the model reduces to the standard GARCH model. If the vectors of autoregressive parameters γ_1, γ_2 are not significantly different from zero, then the variance switches discretely between the values defined in the constant term of the variance equation. In this case, the model reduces to the discrete switching variance of the original Hamilton filter. It would, of course, be possible to find one regime with constant variance and another with a significant GARCH variance process. Once the model is specified, as in (7) and (8), the derivation of the likelihood function follows in the standard way along the lines of (3)-(6).

3 TWO EXAMPLES OF SWITCHING GARCH ESTIMATION

In this section we investigate this more general GARCH model for two series, the three month US treasury bill rate and the spread between three and six month US treasury bills. To give a foretaste of the results, we find that a standard GARCH specification is adequate for the treasury bill rate but that the spread is characterised by a clear break in the variance pattern and is better modelled as a discrete switch in the variance.

The general model we estimate has the following form

$$y_t = B_0^1\xi_t + B_0^2(1-\xi_t) + \sum_{i=1}^{4} y_{t-i}(B_i^1\xi_t + B_i^2(1-\xi_t)) + \omega_t$$

$$\textit{where } \omega \sim NIID(0,\sigma^2_{t\mid t-1}),\ \xi_t \textit{ is the state variableand} \tag{11}$$

$$\sigma^2_{t\mid t-1} = \gamma_0^1\xi_t + \gamma_0^2(1-\xi_t) + (\gamma_1^1\xi_t + \gamma_1^2(1-\xi_t))\omega_{t-1} + (\gamma_2^1\xi_t + \gamma_2^2(1-\xi_t))\sigma_{t-1\mid t-2}$$

and the state equations remain as in (8). This is a general two state switching model which allows a fourth order autoregressive process with a GARCH error process and switching in both the autoregrssive parameters and the parameters in the GARCH process.

3.1 The treasury bill rate

In this section we will examine the US three month treasury bill rate. The raw data for this series is presented in figure 1 which is the data originally used by Hamilton (1988) as an illustration. To the eye there is a clear change of regime between 1979 and 1983 and this was indeed the finding of the switching model. The discrete change in variance during this period is however an important element in identifying the two regimes and we could certainly re-interpret the picture as an autoregressive variance process. We begin to investigate this by first estimating a conventional GARCH model over the whole period and the sub-sample before the apparent structural break. These result are given in table 1.

Table 1:GARCH Models of The Three Month Treasury Bill Rate

	full sample	Sub Sample
γ_0	0.00046(2.5)	0.0002(1.6)
γ_1	0.47(3.4)	0.23(2.1)
γ_2	0.65(8.5)	0.82(10.1)
B_0	0.11(5.2)	0.07(2.1)
B_1	0.91(11.8)	1.03(8.3)
B_2	-0.09(1.1)	-0.23(1.3)
B_3	0.28(3.1)	0.36(2.0)
B_4	-0.2(2.4)	-0.21(1.62)
Log Likelihood	105.15	

There is some sign here of structural instability in that the parameters change quite dramatically between the two periods but the important point to note is that even in the sub-period ending in 1979 there is significant evidence of a GARCH error process. As the standard Hamilton filters out this whole period effectively into one regime this already raises the question of the adequacy of this description. Clearly, however, there is evidence for a structural break at least in the parameters governing the mean of the process and so we will now turn to our general model to see how it performs.

Table 2 gives the results for three models; a version of (11) where the variance is a simple switching constant, a version where the variance is a standard GARCH process and a version where the variance is the fully general switching model as outlined in (11). Figures 1 and 2 also show the data for the treasury bill rate and the regime switching estimated by the three models. All three models detect a very similar structure of regime switching which is also close to that found by Hamilton in his original work. The full general model nests both of the simpler alternatives within it and allows us to test the restrictions through standard likelihood ratio tests. The likelihood ratio test of the restrictions implied by the standard GARCH model is 7.02 which is just within the acceptance region (5% critical value is 7.8). The likelihood ratio test of the restrictions implied by the switching constant variance model is 9.8 which just rejects the restrictions implied by this model(5% critical value is 9.5). Neither the fixed Garch model nor the switching constant model are nested within each other so a standard likelihood ratio test between them is not possible and a non-nested test procedure would need to be used for direct comparisons. However, it is worth noting that the standard GARCH model of table 1 performs almost as well in terms of the likelihood value as the more complicated fixed GARCH with switching parameters model of table 2. We could easily accept the restriction of no switching at all and simply accept the presence of a standard GARCH error process.

In this case both the full-sample and sub-sample estimates find clear evidence of a GARCH error process and when we investigate the possibility of more complex error structures along with general parameter switching we find that the standard GARCH model is an acceptable restriction on the general model while a pure discrete switching model is both rejected by a likelihood ratio test against the general model and is much less parsimonious than the standard GARCH model. It seems, therefore, that in this case a standard GARCH model is a good description of this data series.

Table 2. The Switching GARCH Model for the Treasury Bill Rate.

	General model	Fixed GARCH	Switching constant
γ_0^1	0.008(2.6)	0.01(3.3)	0.15(11.3)
γ_0^2	0.4(2.4)	-	0.56(4.2)
γ_1^1	0.87(6.2)	.86(43)	-
γ_1^2	0.21(1.5)	-	-
γ_2^1	0.00028(0.7)	.000002(1.0)	-
γ_2^2	0.06(1.7)	-	-
B_0^1	1.27(8.4)	1.27(7.4)	2.1(11.8)
B_1^1	0.99(11.1)	1.06(15.1)	1.07(9.6)
B_2^1	-0.04(0.3)	-0.09(1.0)	-0.2(1.1)
B_3^1	0.04(0.3)	0.09(1.1)	0.31(2.0)
B_4^1	-0.11(1.4)	-0.16(3.2)	-0.24(2.2)
B_0^2	1.48(11.3)	1.43(11.9)	0.95(5.3)
B_1^2	-0.55(1.35)	-0.98(5.1)	-0.9(2.9)
B_2^2	0.03(.2)	.65(3.3)	0.001(0.0)
B_3^2	0.22(1.6)	0.33(3.0)	0.22(0.8)
B_4^2	0.20(1.6)	0.06(0.7)	0.12(0.5)
Log Likelihood	109.25	105.74	104.35

Figure 1.

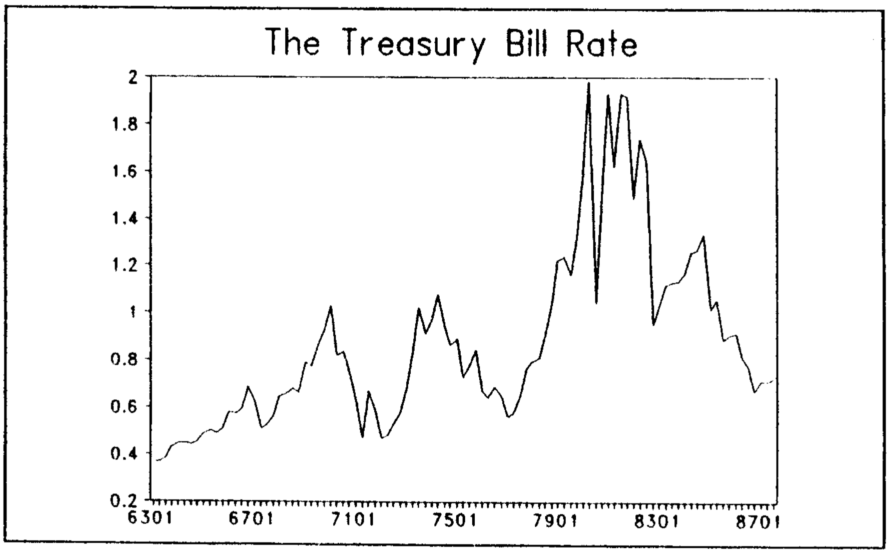
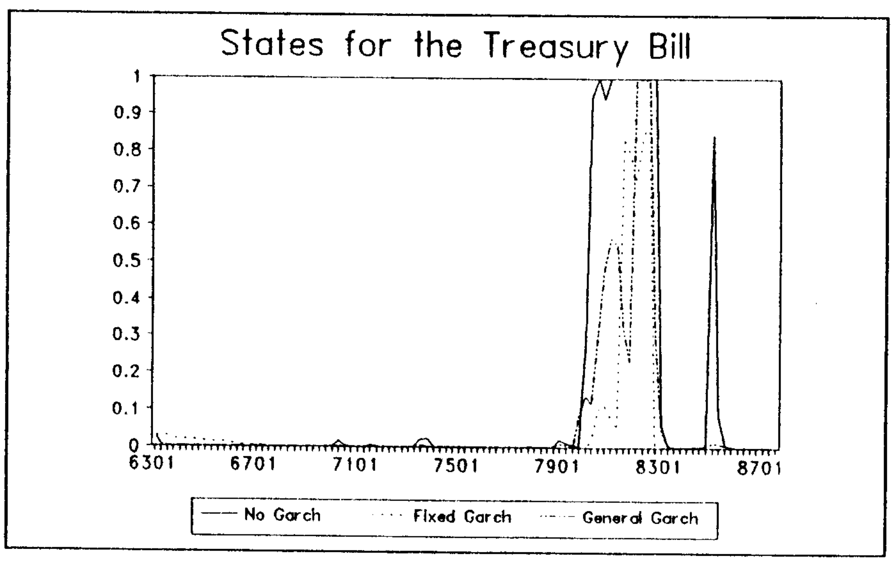


Figure 2.



3.2 The treasury bill spread

In this section we examine the spread between the three and six month US treasury bill rate and the raw data for this series is presented in figure 3. In this case, apart from some large outliers around 1980, there is no obvious signs of a structural break. Although we would suggest that after the outliers we might see some visual evidence of an ARCH effect while before it there is no obvious sign of ARCH effects. We begin to investigate this by again first estimating a conventional GARCH model over the whole period and the sub-sample before the possible structural break. These result are given in table 3.

Table 3:GARCH Models of The Six and Three Month Treasury Bill Spread

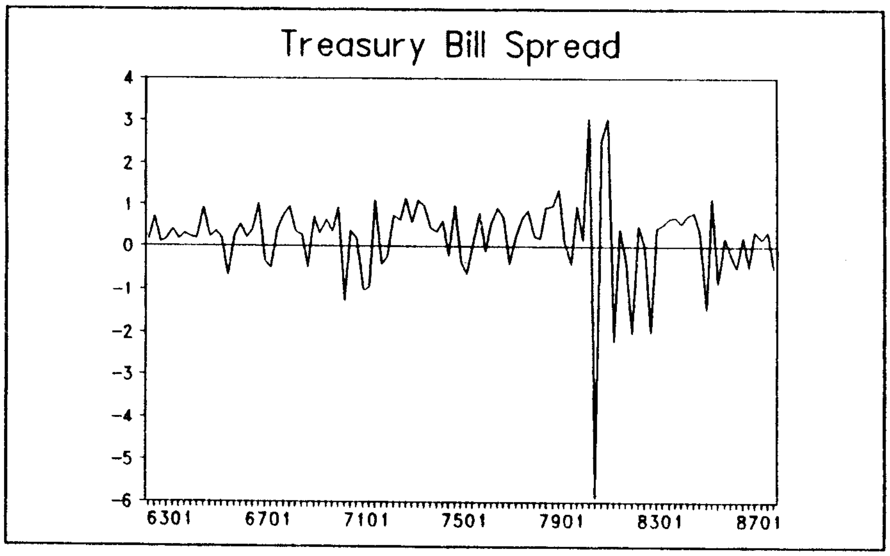
	full sample	Sub Sample
γ_0	0.27(4.0)	0.18(2.3)
γ_1	0.13(3.29)	0.01(1.7)
γ_2	0.65(8.7)	0.0002(0.04)
B_0	0.22(6.5)	0.95(29.8)
B_1	0.22(2.2)	0.02(0.1)
B_2	-0.21(2.5)	-0.03(0.3)
B_3	0.04(0.9)	0.08(0.7)
B_4	0.2(2.2)	0.10(0.8)
Log Likelihood	-17.57	

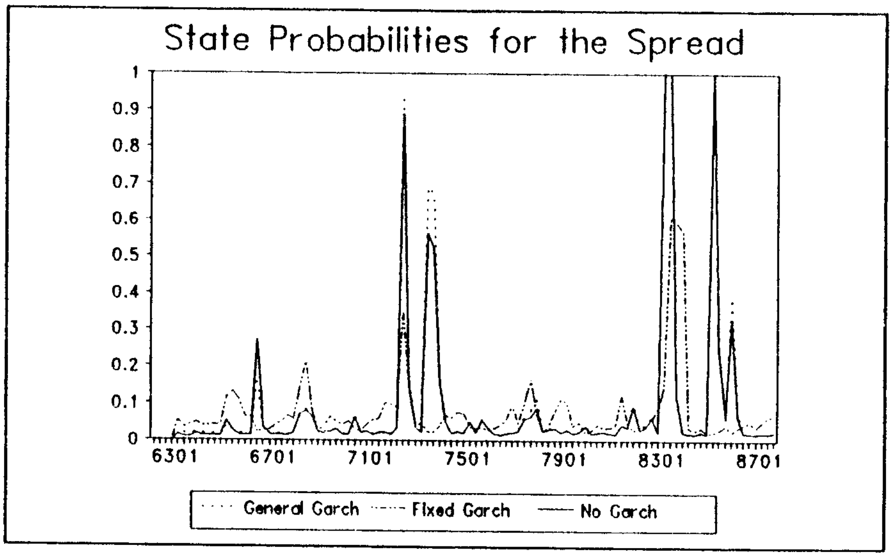
These results contrast quite strongly with those for the three month rate itself. Here we find strong evidence for a GARCH error process over the whole estimation period, but, when we examine only the pre-structural break period all evidence of a conventional GARCH process disappears. So this tends to support our expectation based on a visual inspection of the data that the structural break may lead us to find an overall GARCH process in the whole sample if we estimate a conventional GARCH model.

We now again proceed to test our switching model to see how it performs in this example. Table 4 gives the results for the same three models as table 2. Figures 3 and 4 also show the data for the spread and the regime switching estimated by the three models. The sample separation is not exactly the same between the three models but again it is very similar. The position with respect to the three models is, however, rather different. The likelihood ratio test of the restrictions implied by the standard GARCH model is 31.34 which convincingly rejects the standard GARCH model (5% critical value is 7.8). The likelihood ratio test of the restrictions implied by the switching constant variance model is 4.66 which is easily accepted (5% critical value is 9.5). So, in this case, the discrete switching variance model is a much better description of the data than the standard GARCH model. We are, therefore, able to conclude that the non-switching GARCH model over the full sample finds a significant GARCH effect simply because of a structural break which occurs around 1980. The general switching model detects this break and is able to adequately describe the data with no significant GARCH effects for most of the period.

Table 4. The Switching GARCH Model for the 3 to 6 month Spread.

	General model	Fixed GARCH	Switching constant
γ_0^1	0.012(1.3)	0.1(2.5)	0.43(11.1)
γ_0^2	4.1(2.5)	-	2.06(3.7)
γ_1^1	0.00(0.0)	.99(7.1)	-
γ_1^2	0.28(1.3)	-	-
γ_2^1	0.94(21.4)	.19(1.3)	-
γ_2^2	0.04(0.6)	-	-
B_0^1	-0.43(1.3)	0.05(0.5)	-0.4(1.4)
B_1^1	0.19(1.6)	0.34(3.1)	0.21(1.8)
B_2^1	0.01(0.2)	-0.18(1.9)	0.01(0.1)
B_3^1	-0.03(0.5)	-0.07(1.0)	-0.03(0.4)
B_4^1	-0.03(0.3)	0.2(3.3)	-0.04(0.5)
B_0^2	0.28(5.1)	0.32(4.5)	-0.31(5.1)
B_1^2	-0.62(4.1)	-1.55(5.0)	-0.62(4.6)
B_2^2	-0.1(0.8)	-0.53(1.8)	-0.1(0.8)
B_3^2	0.57(4.8)	0.27(1.5)	0.58(6.1)
B_4^2	0.06(0.4)	-0.4(3.3)	-0.06(0.5)
Log Likelihood	3.87	-11.83	1.54


Treasury Bill Spread
4
3
2
1
0
-1
-2
-3
-4
-5
-6
6301
6701
7101
7501
7901
8301
8701


State Probabilities for the Spread
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6301
6701
7101
7501
7901
8301
8701
General Garch
Fixed Garch
No Garch

4 CONCLUSIONS

This paper has proposed a generalisation of the standard GARCH-M model which allows the whole vector of parameters to switch discretely between two (or potentially more) regimes. Given the recognition that some findings of autoregressive error processes may be due simply to structural breaks in the variance process (Diebold (1986)) this framework becomes a natural one for testing for this possibility. We examine two data sets, the US three month treasury bill rate and the spread between three and six month treasury bill rates. We find that the treasury bill rate does seem to be well characterised by a GARCH error process, although there is evidence for switching in the mean of the process. The spread on the other hand seems to be better described as a discrete switching variance and we may interpret the finding of GARCH effects in the conventional model as being due to the misspecification of the underlying structural breaks. It is perhaps finally worth noting that if the three month treasury bill rate exhibits a GARCH process and the spread between three and six month rates does not, this implies some element of co-movement between the two rates which allows the GARCH process to be cancelled out. This is very similar to the notion of common volatility investigated in Engle and Susmel (1991).

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