Efficient Bailouts?

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October 1, 2012

1 Executive Summary

This paper develops a non-linear DSGE model to assess the interaction between ex-post interventions in credit markets and the build-up of risk ex ante. During a systemic crisis, bailouts to the financial sector relax balance sheet constraints and accelerate the economic recovery. Ex ante, the anticipation of such bailouts leads to an increase in risk-taking, making the economy more vulnerable to a financial crisis. The optimal policy requires in general a mix of ex-post intervention and ex-ante prudential policy. We also analyze the effects of bailouts on financial stability and welfare in the absence of prudential policy. Our results show that the moral hazard effects of bailouts are significantly mitigated by conditioning bailouts on the occurrence of a systemic financial crisis.

A common feature of financial crises is massive government intervention in credit markets. For example, the initial Troubled Assets Relief Program (TARP) required 700 billion dollars to provide credit assistance to financial and non-financial institutions. Related measures in the ongoing European crisis continue to spark an intense debate on the desirability of large scale government intervention. Supporters argue that bailouts are often necessary to prevent a complete meltdown of the financial sector, which would in an extraordinary contraction in output and employment. Critics argue that bailouts create incentives for investors to take even more risk ex ante, sowing the seeds for future crises. These critics propose regulations to limit the central bank's ability to bailout the financial sector.

How does the expectation of a bailout impact the stability of the financial sector? Is it desirable to prohibit the use of public funds to bailout the financial sector? How large should bailout packages be? How critical are policies to prevent excessive risk-taking?

This paper contributes to the above debate by proposing a quantitative framework to evaluate the impact of bailouts on the economy. Previous studies on this subject have been confined to stylized three period models. The trade-offs involved in this matter, however, require a quantitative model where a formal welfare based analysis can be conducted. This paper attempts to fill this gap in the literature.

Our analysis yields the following results:

- Optimal policy requires a combination of bailouts and prudential policy

- Systemic bailouts *alone* are ex-ante welfare improving despite moral hazard effects
- Idiosyncratic bailouts generate significant increase in build-up of risk and are welfare *reducing*
- The optimal bailout is approximately 2 percentage points of GDP and it is increasing in leverage
- The severity of recessions falls by 40% with the optimal intervention item When bailouts are unanticipated the optimal intervention is reduced by half and the recession also becomes much less severe

2 Introduction

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This paper addresses these questions using on a non-linear DSGE model in which credit frictions generate scope for bailouts during a financial crisis, but where the anticipation of bailouts generate more risk-taking before the crisis actually hits. Recent research (e.g. Gertler and Kiyotaki, 2010) has analyzed how credit policy can mitigate a credit crunch and the resulting recession ex-post. At the same time, a growing theoretical literature investigates the moral hazard implications of large scale government interventions (e.g. Farhi and Tirole, 2010). However, there has been little work assessing the quantitative implications of moral hazard. The goal of this paper is to develop a quantitative DSGE model to assess the interaction between ex-post interventions in credit markets and the build-up of risk ex ante in a unified framework. We use this quantitative framework to derive the optimal intervention in the economy and evaluate the macroeconomic and social welfare effects.

The model features a representative corporate entity that faces two frictions in its capacity to finance investment. First, debt contracts are not fully enforceable, giving rise to a collateral constraint that limits the amount that firms can borrow. Second, there is an equity constraint that imposes a minimum dividend payment that firms must make each period. Therefore, firms balance the desire to increase borrowing and investment today with the risk of becoming financially constrained in the future. When leverage is sufficiently high and an adverse financial shock hits the economy, firms are forced to cut down on investment, leading to a protracted recession.

In our model, credit crunches are socially inefficient because firms remain undercapitalized, hindering the economic recovery. From an individual point of view, households do not have an incentive to unilaterally transfer funds to firms, since this only entails costs for them. From a social point of view, a bailout by the government represents a Pareto improvement because the collective transfer to firms allows all households to obtain higher dividends and higher labor income. We derive the optimal form of intervention by considering a social planner that is subject to the same constraints as the private economy. To capture efficiency costs from government intervention, we assume that there is an iceberg cost associated with transfers from households to firms. This implies that the government finds it optimal to conduct bailouts only when the credit crunch is sufficiently severe.

The planner's intervention results in financial constraints becoming frequently more binding in the economy although its effects in the economy are much less severe. In fact, we find that the the impact loss in output that results from a credit crunch falls 20 percent under the optimal intervention. Moreover, the recession that follows crises become less protracted resulting in significantly smaller cumulative output losses. Hence, while the planner's intervention induces more risk-taking, it does not cause more financial fragility due to the fact that the planner can alleviate the effects of adverse financial shocks.

We analyze different decentralizations for the optimal intervention. The key feature of ex-post interventions is that they imply a redistribution from households to firms. Accordingly, policies like debt relief and equity injections are effective during a systemic crisis. Importantly, the precise form of the bailout package has implications for the need of ex-ante intervention.

In general, the optimal policy mix requires a combination of bailouts and prudential policy. For the case of a fully non-targeted bailout, however, bailouts are enough to implement the constrained planner's allocations. This arises because by fully indexing bailouts to aggregate variables, individual financial decisions are not distorted. In a more realistic case in which bailouts are targeted, i.e., bailouts interact with individual financial decisions, the government requires a prudential tax on debt to implement the planner's allocations. We find, however, that if targeted bailouts are implemented only in systemic financial crisis, there are still strictly positive welfare gains from conducting only bailouts, and the magnitudes of the gains come quite close to the gains with the fully optimal policy. We emphasize that these welfare gains occur not only ex-post but also ex-ante. On the other hand, if bailouts are conditioned only in individual financial decisions, this causes a sharp increase in financial fragility and substantial ex-ante welfare losses.

Related Literature — This paper draws on the extensive literature on the macroeconomic effects of financial frictions, shaped by the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In particular, our model shares with Jermann and Quadrini (2012) the emphasis on financial shocks and equity financing decisions, and with Mendoza (2010) the emphasis on non-linear dynamics beyond the steady state.[1] However, these papers do not address normative issues.

This paper is also related to a growing quantitative literature that studies the effects of credit policy during a credit crunch (e.g. Gertler and Karadi (2011), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010)).[2] For reasons of tractability, most of this literature focuses on policy measures in response to unanticipated crises or on log-linear dynamics around the deterministic steady state and does not address risk considerations and the moral hazard effects of credit policy. Instead, a distinctive feature of this paper is the consideration of how expectations of future bailouts affect ex-ante risk-taking. This is crucial to assess the dynamic implications of credit intervention on financial stability and on social welfare.

The recent work by Gertler, Kiyotaki, and Queralto (2011) is more closely related to our paper. Gertler et al. develop a model in which banks have access to debt and equity financing and investigate the moral hazard effects of credit policy. They focus on the macro dynamics around a “risk-adjusted" steady state in which financial constraints are always binding. Our paper differs in two important aspects. First, we characterize and solve for the optimal bailout policy and prudential policy to avoid excessive risk-taking. Second, we conduct our analysis using a global solution method and study full equilibrium dynamics in a stochastic steady state in which binding financial constraints are rare events. On the other hand, we also acknowledge that our framework is not as flexible as Gertler et al. to handle richer features considered in larger scale DSGE models.

[1]Other recent studies of business cycles that emphasize non-linear dynamics include He and Krishnamurthy (2012) and Perri and Quadrini (2011).

[2]Other recent models of credit crunches include Guerrieri and Lorenzoni (2011), Bigio (2010), Midrigin and Philippon (2011), Shourideh and Zetlin-Jones (2012).

This paper also builds on the theoretical literature that analyzes the incentive effects of bailouts on financial stability. Farhi and Tirole (2010) and Chari and Kehoe (2009) emphasize how bailouts can increase financial fragility and draw implications for ex-ante regulation. Schneider and Tornell (2004), Diamond and Rajan (2009) and Keister (2010) emphasize, as we do, that bailouts can be optimal ex-ante as a form of insurance, but their focus is mostly on self-fulfilling crises.[3] Our analysis mainly differs by developing a quantitative framework to assess the macroeconomic and welfare effects of bailouts.

This paper is also related to a growing quantitative literature on how macro-prudential policy can be used to reduce the level of financial fragility.[4] Our paper also gives a role to limits on leverage during good times, but, unlike this literature, it arises as a result of an externality created by bailouts. Moreover, their inefficiency relates to the effects between the intertemporal reallocation of wealth across leveraged borrowers and equilibrium prices, while here it is related to the effects between the intratemporal reallocations of wealth between shareholders and managers of corporations and investment capacity.

The remainder of the paper is organized as follows: Section 2 presents the analytical framework; Section 3 analyzes the optimal intervention; Sections 4 and 5 present the quantitative analysis; and Section 6 discusses the conclusions.

[3]See also Keister and Narasiman (2011) for recent work on how policy issues might be affected by whether a financial crisis is originated by fundamentals or self-fulfilling expectations.

[4]See e.g. Bianchi (2011), Bianchi and Mendoza (2010), and Jeanne and Korinek (2010). Benigno, Chen, Otrok, Rebucci, and Young (2012) also discuss ex-post policy measures to address a pecuniary externality (see also Jeanne and Korinek (2011)). For theoretical contributions see e.g. Lorenzoni (2008) and Caballero and Krishnamurthy (2001).

3 Analytical Framework

Our model economy is a small open economy populated by firms and workers who are also the firms' shareholders. We begin by describing the decisions made by different agents in the economy, and then we discuss the general equilibrium.

3.1 Households

There is a continuum of identical households of measure one that maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(n_t)) \tag{1}$$

where c_t is consumption, n_t is labor supply, β is the discount factor, and $G(\cdot)$ is a twice-continuously differentiable, increasing and convex function. The utility function $u(\cdot)$ has the constant-relative-risk-aversion (CRRA) form; the composite of the utility function has the GHH form, eliminating wealth effects on the labor supply. The advantage of these preferences is that they deliver realistic responses of employment during a credit crunch without introducing frictions in labor markets that would complicate the analysis.

Households do not have access to bond markets, and they are the firms' shareholders. This yields the following budget constraint:

$$s_{t+1} p_t + c_t \leq w_t n_t + s_t(d_t + p_t) \tag{2}$$

where s_t represents the holdings of firm shares and p_t represents the price of firm shares.

The first-order conditions are given by:

$$w_t = G'(n_t) \tag{3}$$

$$p_t u'(t) = \beta \mathbb{E}_t u'(t+1)(d_t + p_{t+1}) \tag{4}$$

Iterating forward on (4) and imposing a no-bubble condition yields that in equilibrium, the price of shares must be equal to:

$$p_t = \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j m_{t+j} d_{t+j} \tag{5}$$

where $m_{t+j} \equiv (\beta^j u'(c_{t+j} - G'(n_{t+j})))/(u'(c_t - G'(n_t)))$ represents the stochastic discount factor.

3.2 Corporate Entities

There is a measure one of identical firms with technology given by the production function $F(z_t, k_t, h_t)$ that combines capital denoted by k, and labor denoted by h to produce a final good. TFP denoted by z_t follows a first-order Markov process. Consistent with the typical timing convention, k_t is chosen at time $t-1$ and is therefore predetermined at time t. Instead, the input of labor h_t can be flexibly changed in period t.

Firms have the following technology to transform final goods into investment goods.

$$k_{t+1} = k_t(1-\delta) + i_t \tag{6}$$

where i_t is the level of investment and δ is the depreciation rate. Capital accumulation is subject to convex adjustment costs, given by $\psi(k_t, k_{t+1})$. Adjustment costs are introduced to improve the quantitative performance of the model in terms of the volatility of investment and asset pricing implications.

Firms pay dividends, denoted by d_t, and issue non-state contingent debt, denoted by b_{t+1}. Firms finance investment, including capital adjustment costs ($i_t + \psi(k_t, k_{t+1})$), debt repayments (b_t), dividend payments (d_t) with internal cash flows ($F(z_t, k_t, h_t) - w_t n_t$), and new debt ($b_{t+1}$). The flow of funds constraint for firms is then given by:

$$b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq F(z_t, k_t, h_t) - w_t n_t + \frac{b_{t+1}}{R_t} \tag{7}$$

where w_t is the wage rate, and R_t is the gross interest rate determined exogenously in international markets. R_t is stochastic and follows a first-order Markov process. Implicit in the flow of funds constraint is the fact that firms cannot issue new shares (we normalize the total number of shares to 1). However, they can adjust retained earnings by cutting dividend payments and servicing debt.[5]

Firms face two types of liquidity constraints on their ability to finance investment. First, they are subject to a collateral constraint that limits the amount of borrowing to a fraction of their capital holdings:

$$b_{t+1} \leq \kappa_t k_{t+1} \tag{8}$$

This constraint is similar to those used in existing literature, and we interpret it as arising in an environment where creditors can only recover a fraction κ_t of the firms' assets.[6] Following Jermann and Quadrini (2012), κ_t represents a shock to the borrowing capacity of firms. For simplicity, this "financial shock" follows a two-state Markov chain with values given by κ^H and κ^L. In our quantitative analysis, the collateral constraint will never bind when $\kappa = \kappa^H$, so that when κ switches from high to low, this may lead to a binding constraint and a credit crunch. Note that whether the economy enters a credit crunch depends endogenously on the degree of leverage in the economy.

Without any constraints on equity financing, the shadow value of external funds would be equal to one. We assume that there is a lower bound on dividends given by $\bar{d}$, i.e., at each period firms are required to satisfy:

$$d_t \geq \bar{d} \tag{9}$$

A special case is the restriction that dividends need to be non-negative, which effectively implies that the issuance of new shares is not available. This constraint reflects the notion that dividend payments are required to reduce agency problems and information asymmetries between shareholders and managers.

We assume that firms maximize shareholder value as is standard in the dynamic corporate finance literature. Maximization of shareholder values implies that firms must discount profits at state $t+j$ the rate m_{t+j} defined above. That is, their problem is to maximize $\mathbb{E}_t \sum_{j=0}^{\infty} m_{t+j} d_{t+j}$.

[5]We have assumed for simplicity that firms do not trade used capital, because this does not occur in equilibrium as all firms are identical.

[6]We implicitly assume that the liquidation value of capital is not affected by market prices, thereby turning off a fire-sale externality mechanism (see e.g. Bianchi and Mendoza (2010) for an analysis of this channel). We make this assumption to focus on an alternative mechanism which results from reallocation of funds between shareholders and firms.

3.3 Recursive Problem and Optimality Conditions

The aggregate state vector of the economy that we denote by X is given by the aggregate level of capital K, bonds B, and the aggregate shocks κ and z, i.e., $X = K, B, \kappa, z, R$. Denoting $V(k,b,X)$ the cum-dividend market value of the firm and using prime to denote next period variables, the optimization problem for firms can be written recursively as:[7]

$$V(k,b,X) = \max_{d,h,k',b'} d + \mathbb{E}m'(X,X')V(k',b',X') \tag{10}$$

$$s.t.$$

$$b + d + k' + \psi(k,k') \leq (1-\delta)k + F(z,k,h) - wn + \frac{b'}{R}$$

$$b' \leq \kappa k'$$

$$d \geq \bar{d}$$

The optimality condition for labor demand yields a standard static condition:

$$F_h(z_t, k_t, h_t) = w_t \tag{11}$$

There are also two Euler intertemporal conditions that relate the marginal benefit from distributing one unit of dividends today with the marginal benefit of investing in the available assets and distributing the resulting dividends in the next period. Denoting by μ, the multiplier associated with the borrowing constraint, η the multiplier associated with the dividend payout constraint, the Euler equations and associated complementary slackness conditions are given by:

$$1 + \eta_t = R_t \mathbb{E}_t m_{t+1}(1 + \eta_{t+1}) + R_t \mu_t \tag{12}$$

$$(1+\eta_t)(1+\psi_{2,t}) = \mathbb{E}_t m_{t+1}\left[1 - \delta + F_{k,t+1} - \psi_{1,t+2}\right](1+\eta_{t+1}) + \kappa_t \mu_t \tag{13}$$

$$\mu_t(\kappa_t k_{t+1} - b_{t+1}) = 0, \;\; \mu_t \geq 0 \tag{14}$$

$$\eta_t(d_t - \bar{d}) = 0, \;\; \eta_t \geq 0 \tag{15}$$

[7] We note that since the firm is representative, in equilibrium the firm never exposes itself to the possibility of not being able to satisfy the financial constraints. Hence, there is no voluntarily or involuntary default.

In the absence of financial constraints on borrowing and dividend payments, the cost of raising equity (by reducing dividends), i.e., $1/\mathbb{E}_t m_{t+1}$, would be equal to the cost of debt R_t, and firms would be indifferent at the margin between equity and debt financing. However, when the collateral constraint binds, there is a wedge between the marginal benefit of borrowing one more unit and distributing it as dividends in the current period and the marginal cost of cutting dividends in the next period to repay the debt increase. In addition, when the dividend payout constraint binds, a positive wedge arises between the marginal benefit from investing one more unit in capital or bonds and the marginal cost of cutting one unit of dividends.

As we will see in the quantitative analysis, the collateral constraint and the dividend payout constraint often bind at the same time. Intuitively, both constraints impose a limit on a firm's funding ability. A binding dividend payout constraint forces higher levels of borrowing for given investment choices. Similarly, a tighter constraint on borrowing puts pressure on the firms to finance with equity. Note that in equilibrium, reducing dividend payments increases the cost of equity because households have a concave utility function.

Discussion of Financial Frictions — Our normative analysis requires a model of incomplete markets that departs from Modigliani-Miller. We discuss now the specific assumptions that we have made to deviate from Modigiliani-Miller results.

As we will show below, the key friction in our setup is that firms face an equity constraint that imposes a lower bound on dividend payments. This is a relatively standard way of capturing agency problems and information asymmetries between a firm's shareholders and its managers, and it is in line with an extensive literature documenting the importance of agency frictions between shareholders and corporate managers (see .e.g. Shleifer and Vishny (1997) for a survey). Without this constraint on dividend payments, firms would be able to raise enough equity to finance desired investments and would fail to reproduce the evolution of real and financial variables in the data.[8]

Borrowing by firms is limited by imperfect enforceability of contracts. In particular, we assume that creditors require firms to hold collateral to back promised repayments according to (8). In

[8]Several papers, including Jermann and Quadrini (2012) and Gilchrist, Sim, and Zakrajšek (2010), follow similar assumptions. More generally, what is necessary is to assume that equity becomes relatively costly to issue in bad states of nature. One motivation for this frictions involves the lack of incentives to issue equity when managers have private information about investment opportunities. For example, in Myers and Majluf (1984), good firms may find it optimal not to issue equity when they are pooled with those of lower quality (see also Bigio (2011) for recent work linking adverse selection in credit markets with banks' equity financing).

order to enrich the model, we introduce shocks to how much collateral firms firms are required to pledge. A possible interpretation of this shock relates to disruptions in financial intermediaries, which become either less willing to lend or more concerned about the riskiness of the corporate sector. We will show that when leverage is sufficiently high, a negative financial shock produces a credit crunch with similar features to the data. Jermann and Quadrini (2012) recently pointed out that financial shocks improve the quantitative performance of a business cycle model.

We have also assumed that asset markets are restricted to one-period non-state contingent bonds, as is standard in the literature and represents a simplification of the limited insurance that firms have access to. As long as hedging is costly, our qualitative results would continue to hold with contingent debt. What is important for our results is that firms cannot fully undo the equity constraints using contingent debt.[9] Finally, households in our model do not have access to credit markets. Households can, however, smooth consumption through dividend payments, which in turn will affect firms' financial decisions.[10]

Overall, these assumptions allow us to formulate a parsimonious analysis of optimal bailouts. Moreover, these assumptions are important for the model to produces financial and real flows that are broadly consistent with key features of the data in terms of general co-movements and financial crises dynamics.

3.4 Competitive Equilibrium

The competitive equilibrium for a small open economy that borrows from abroad at an exogenous interest rate can be constructed in the usual form. Market clearing in the labor market requires:

$$h_t = n_t \tag{16}$$

[9]Standard motivations for restrictions on state-contingent liabilities are the lack of commitment by investors to inject funds to the firms in bad times or the inability to verify the realization of the shocks. We do not model these frictions, though.

[10]At the computational cost of introducing an additional state variable, this constraint can be relaxed to some extent. What is important is that households do not have access to perfect credit markets in order to guarantees that firms do not have an incentive to deleverage in the long-run. Notice that if $\bar{d} = -\infty$, and if households have *unrestricted* access to international credit markets by borrowing and saving at the interest rate R_t, the competitive equilibrium would be unaffected by financial shocks. Combining the household's first-order condition, ($R_t E m' = 1$) and the firm's first-order condition, ($R_t E m' + \mu = 1$) would yield $\mu = 0$. Therefore, the Modigiliani Miller theorem would hold, and the model would become a standard RBC model.

Market clearing in equity markets requires:

$$s_t = 1 \tag{17}$$

Using the two equations above and combining the household budget constraint and the firms' flow of funds constraint, we obtain the resource constraint for the economy:

$$b_t + c_t + k_{t+1} + \psi(k_t, k_{t+1}) = (1-\delta)k_t + F(z, k_t, h_t) + \frac{b_{t+1}}{R_t} \tag{18}$$

The recursive competitive equilibrium can be defined as follows:

Definition 1 *A recursive competitive equilibrium is given by (i) firms' policies $\{\hat{d}(k,b,X), \hat{h}(k,b,X), \hat{k}(k,b,X), \hat{b}(k,b,X)\}$; households's policies $\hat{s}(s,X), \hat{c}(s,X), \hat{n}(s,X)$; a stochastic discount factor $m(X,X')$; firm's value $V(k,b,X)$; prices $w(X), p(X)$; and a law of motion of aggregate variables $X' = \Gamma(X)$, so that: (i) households solve their optimization problem; (ii) firms' policies and firms' value solve (10); (iii) markets clear in equity market ($\hat{s}(1,X) = 1$) and labor market ($\hat{h}(K,B,X) = \hat{n}(1,X)$), (iv) the stochastic discount factor for firms is given by the household's marginal rate of substitution $m(X,X') = \beta u'(\hat{c}(1,X) - G(\hat{n}(1,X)))/(u'(\hat{c}(1,X') - G(\hat{n}(1,X')))$ (v) the law of motion $\Gamma(\cdot)$ is consistent with individual policy functions and stochastic processes for κ and z.*

Necessary and sufficient conditions for a competitive equilibrium can be established due to the fact that the optimization problem for households and firms are convex programs. In particular, given stochastic processes for R_t, z_t and κ_t, a set of stochastic sequences $\{c_t, k_{t+1}, i_t, b_{t+1}, d_t, h_t, n_t, w_t, p_t, \mu_t, \eta_t, s_t\}_t$ is a competitive equilibrium if and only if equations (3)-(4) and (6)-(18) are satisfied.

In order to illustrate the properties of the model, it is useful to first analyze the case without uncertainty. In a deterministic steady state with $\beta R < 1$, (i) the collateral constraint is always binding, and (ii) there exists $\hat{d}$ such that the dividend payout constraint binds if $\bar{d} > \hat{d}$. For (i), note that in a deterministic steady state, $m_t = 1$ and (12) is simplified to $1 = \beta R + \mu$. Since $\beta R < 1$, this implies that $\mu > 0$. For (ii), one can obtain the steady state values $[k^{ss}, h^{ss}, b^{ss}, \mu^{ss}]$ from (3),(8),(12), and (13). Substituting these expressions, (8) and (3) in the flow of funds constraint (22) yields the value of dividends at steady state $d^{ss} = F(z^{ss}, k^{ss}, h^{ss}) - k^{ss}(\delta + \bar{\kappa}(R-1)/R) - h^{ss}G'(h^{ss})$.

In general, in a stochastic steady state, these financial constraints may or may not bind depending primarily on the magnitude of the shocks, βR, and the tightness of the constraints.

4 Normative Analysis

4.1 Scope for Policy

We start the normative analysis by discussing the scope for government intervention in our setup. The key externality in the model is related to the undercapitalization of firms. Households are not willing to unilaterally transfer funds to firms because they only incur costs. This is a form of a free-rider externality. Instead, a social planner recognizes that transferring resources to the firm increases labor payments and dividend payments in future periods for all households in the economy.

As in the Holmstrom-Tirole analysis of liquidity, there is a rationale for the government to transfer resources from consumers to producers. Three comments are in order. First, the ability of the government to improve welfare relies on the ability to extract payments from households via taxes to address the free-rider externality. The government, however, does not have a superior debt capacity than the private sector. That is, the government does not use public debt as private liquidity Woodford (1990). Second, in our setup, because households own firms, transfers will lead to Pareto improving transfers. Finally, to reflect the fact that these transfers are costly in practice, we assume that there is an iceberg cost φ on the volume of transfers. This cost could arise from distortions in taxation or the financing of inefficient projects. However, we do not model the causes of the cost explicitly.[11]

4.2 Constrained Social Planner's Solution

We consider a benevolent social planner who (a) directly chooses the sequence of debt, capital, and equity payout subject to the liquidity constraints and the resource constraint; (b) chooses a

[11] For the sake of simplicity, we assume that the cost is linear on the transfer to each firm, but we could allow for a more general cost. For example, we could introduce a fixed cost to bail out a single firm, implying that the government would provide a bailout when there is a sufficient number of firms in distress (see Farhi and Tirole (2010) for a theoretical analysis and Kelly, Lustig, and Van Nieuwerburgh (2011) for empirical evidence on the anticipation of systemic bailouts). Since all firms are identical, the implications for the social planner solution would be very similar. We will come back to these issues when we address implementability.

sequence of transfers Υ_t between firms and households at a linear cost φ; (c) lets labor markets, stock market and goods markets clear competitively.

It is necessary to mention a few factors in our formulation of the planner's problem. First, by making the planner subject to the same financial constraints as the decentralized equilibrium, the economy is also subject to the deleveraging effects of financial shocks. Finally, while the social planner cannot directly affect labor market outcomes, it may affect the labor market indirectly through the choice of capital as it affects wages in the next period.

Denote by $\Upsilon_t \geq 0$ the transfer from households to firms, and $\hat{w}_t(k,z)$ and $\hat{h}_t(k,z)$, the equilibrium wage and labor allocations. Since labor is chosen by households and firms in competitive markets, wages and labor satisfy (3) and (11). The problem of the social planner can be written as follows:

$$\max_{\{k_{t+1},b_{t+1},c_t,p_t,\Upsilon_t\geq 0\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(\hat{h}_t(k_t,z_t)))$$

$$\begin{aligned}
(1-\delta)k_t + F(z_t,k_t,h_t) + \frac{b_{t+1}}{R_t} - b_t - k_{t+1} - \psi(k_t,k_{t+1}) - \varphi\Upsilon_t = & \quad c_t \\
(1-\delta)k_t + F(z,k_t,h_t) - \hat{w}_t(k_t,z_t)\hat{h}_t(k_t,z_t) + \frac{b_{t+1}}{R_t} + \Upsilon_t - b_t - k_{t+1} - \psi(k_t,k_{t+1}) & \quad \geq \bar{d} \\
b_{t+1} \leq & \quad \kappa_t k_{t+1} \\
\beta \mathbb{E}_t u'(t+1)(d_{t+1} + p_{t+1}) = & \quad p_t u'(t)
\end{aligned}$$

where $\hat{w}_t(k_t,z_t) = G'(\hat{h}_t(k_t,z_t)) = F_L(z_t,k_t,\hat{h}_t(k_t,z_t))$. We attach again η_t and μ_t to the financial constraints. To facilitate comparison with the conditions characterizing the competitive equilibrium, we normalize these multipliers by the marginal utility of consumption. Notice that the last condition does not bind, as the price of shares do not affect the set of feasible allocations for the planner.

First-order condition with respect to Υ_t yields:

$$\varphi \geq \eta_t \text{ with equality if } \Upsilon_t > 0 \tag{19}$$

Condition (19) is crucial to asess the tradeoffs involved in the bailout policy at the margin. This condition establishes that the planner will transfer resources from households to firms until the marginal cost given by φ equals the marginal benefits, given by the shadow value from relaxing the equity constraint η_t (expressed in units of consumption). It also follows that $\Upsilon_t = 0$ if the equity constraint is not binding or if the shadow value from relaxing the dividend payout constraint is small enough. Note that it is not optimal to fully relax the dividend payout constraint, i.e. if $\Upsilon_t > 0$ for some t, it also follows that $\eta_t > 0$. We also have the following two results:

Corollary 1 *If $\varphi = 0$, the equity constraint does not bind for the social planner.*

Proof: Setting $\Upsilon_t > \bar{d} + b_t + i_t + \psi(k_t, k_{t+1}) - F(z_t, k_t, n_t) - w_t h_t + \frac{b_{t+1}}{R_t}$, the planner can completely relax the equity constraint without affecting the objective function or the rest of the constraints. Intuitively, if taxes are not distortive, the planner can use cost-free transfers as a substitute for lower dividend payments when the dividend payout constraint becomes binding.

Corollary 2 *If $\bar{d} = -\infty$, the competitive equilibrium and the social planner's solution coincide.*

Proof: The proof notes that $\bar{d} = -\infty$ implies $\Upsilon_t = 0$ and $\eta_t = 0$, which yields that the conditions characterizing the competitive equilibrium are identical to those characterizing the social planner. Since firms have unrestricted access to equity, implementing a transfer from households to firms has no effect. It also follows that if $\bar{d} = -\infty$ and $\varphi > 0$, bailouts are welfare reducing.

In the policy experiment below, we consider the general case where $\varphi > 0$, which implies that the government faces strictly positive efficiency costs from transferring resources from firms and households. Under these conditions, bailouts involve ex-post a trade-off between relaxing firms' balance sheet constraints and the iceberg costs associated with the transfer.

First-order condition with respect to capital and using the equilibrium conditions for the labor market yields:

$$(1+\eta_t)(1+\psi_{2,t}) = \frac{\beta \mathbb{E}_t u'(t+1)}{u'(t)} \left[1 - \delta + F_{k,t+1} - \psi_{1,t+2}\right](1+\eta_{t+1}) \left(1 - \frac{\partial \hat{w}_{t+1}}{\partial k_{t+1}} \hat{h}_{t+1}\right) + \kappa_t \mu_t \tag{20}$$

An important difference between (20) and the analogous condition for firms (13) is that the planner internalizes how next period capital stock affects next period wages, which in turn affects the tightness of the equity constraint. In particular, firms do not internalize that one more unit of capital tightens the constraint by $\frac{\hat{w}_{t+1}}{\partial k_{t+1}}\hat{h}_{t+1}$ which has a marginal utility cost of η_{t+1}.

4.3 Decentralization

This section analyze possible decentralization to the social planner's allocations. As we will see, the decentralization requires in general both ex-ante prudential measures and ex-post policy measures.

Debt Relief — We first analyze the role of debt relief. We consider a policy in which the government pays a fraction γ_t of private debts and finances this transfer of funds and its iceberg cost with lump sum taxes T_t to households. In addition, the government sets taxes on borrowing and capital τ_t^b and τ_t^k that is rebated by a lump-sum tax to firms, given by T_t^f. With these policies, the households' budget constraint and the firms's flow of funds constraint become respectively:

$$s_{t+1}p_t + c_t \leq w_t n_t + s_t(d_t + p_t) - T_t \tag{21}$$

$$(1-\gamma_t)b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq (F(z_t, k_t, h_t) - w_t n_t)(1-\tau^k) + \frac{b_{t+1}}{R_t}(1-\tau_t) + T_t^f \tag{22}$$

First-order condition with respect to b_{t+1} yields:

$$1 + \eta_t = R_t(1+\tau_t)\mathbb{E}_t m_{t+1}(1+\eta_{t+1})(1-\gamma_{t+1}) + R_t(1+\tau_t)\mu_t \tag{23}$$

The rest of the optimality conditions remain the same. Note that from (23), the private costs of borrowing at time t are reduced by a factor of $(1-\gamma_{t+1})$ in a state t+1 in which the government provides debt relief. An examination of these first-order conditions leads to the following proposition:

Proposition 1 *The government can implement the constrained optimal allocations through a combination of debt relief, taxes on debt, and lump sum taxes. These polices are given by:*

$$\gamma_t = \frac{\Upsilon_t}{b_t}, \quad T_t = \Upsilon_t(1+\varphi), \quad T_t^f = \frac{b_{t+1}}{R_t}\tau_t$$

$$\tau_t^b = \frac{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1})(1-\gamma_{t+1}) + \mu_t} - 1, \tau_t^k = \frac{\mathbb{E}_t m_{t+1}\left(\frac{\partial \hat{w}_{t+1}}{\partial k_{t+1}}\hat{h}_{t+1}\right)\eta_{t+1} + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t} - 1$$

where all variables are evaluated at the constrained optimal allocations.

The proof notes that the specified policy instruments demonstrate that the conditions characterizing the competitive equilibrium are identical to those of the constrained optimal allocations (see Propositions 1 and 2).

The role of the tax on debt is Pigouvian, as it aims to correct the private cost of borrowing, which is distorted by debt guarantees. Note that the tax on debt is only strictly positive when debt relief are activated with strictly positive probability in the next period, an event that occurs when the dividend payout constraint becomes binding in the economy. Hence, the tax on debt is prudential.

Lump-sum transfers — The final policy instrument we consider is a lump-sum transfer that is independent of any individual choice made by the firms. In contrast to the above polices, first-order conditions are unaffected when the government bails out firms with a lump-sum transfer, which implies that no prudential policy is required. Intuitively, the government provides insurance in such a way that firms see the benefits from the bailout as entirely exogenous from their financial decisions. As a result, the government can implement the constrained optimal allocations without resorting to a prudential tax on debt. It is important to keep in mind that lump-sum transfers are extremely impractical for reasons that are not modeled in this paper In this respect, we see the implementation of lump-sum transfers as purely illustrative.

Financial Intermediaries — In practice, central banks implement a variety of policies with the aim of facilitating the corporate sector's access to credit, some of which involve bailouts to financial institutions. We would like to point out that both the prudential and ex-post policy measures we have described can also be applied to financial intermediaries in a simple extension of our model.[12] To simplify the analysis, we do not model financial intermediaries and consider only direct bailouts

[12]In particular, one could modify the problem of the corporate entities so that instead of carrying out production activities on their own, they provide intra-period loans to firms in the form of capital goods. It is easy to see that this modified setup would be isomorphic to our baseline model. As a result, one could reinterpret the model as applied to the banking sector. See also Bianchi (2011) for a mapping between taxes on debt, capital requirements, and margin requirements.

to firms. The crucial factor for our analysis is that this intervention relaxes balance sheets across the economy and mitigates the fall in credit and investment that occurs during crises.

4.4 Quantitative Policy Experiments

4.5 Quantitative Policy Experiments

In our quantitative analysis, we will start by exploring the properties of the optimal policy instruments and its effects over macroeconomic dynamics. As we analyzed above, the optimal policy requires, in general, prudential policy to counteract excessive risk-taking incentives. We are also interested in examining the importance of the complementarity between bailouts and prudential policy. For this purpose, we will analyze two additional policy experiments. One experiment consists of imposing the bailout policy— implemented with debt relief— we derive from the optimal intervention, but without the use of prudential policy. We call this policy systemic bailout policy. Second, we study idiosyncratic bailouts. In this scenario, bailouts now depend entirely on firm specific choices independently on aggregate states. In particular, the government uses the debt relief policy solved above, but now the bailout is given by $\Upsilon(b_t, k_t, z, \kappa)$, i.e. there is no subscript in Υ associated with macro variables.

These two additional experiments allows us to analyze the trade-off involved in bailouts. Ex-post, bailouts help relax balance sheet constraints. Ex ante, there is too much risk taking relative to the social optimum. Hence, we will study whether is is possible to increase welfare using bailouts without prudential policy.

4.6 Numerical Solution

The numerical solution to the model involves several challenges. First, there are the well-known complications of non-linearities that arise in incomplete markets and, in particular, the occasionally binding financial constraints. In addition, the state variables in the model are not confined to a narrow region of the state space. We use a policy function iteration algorithm whereby we approximate the equilibrium functions over the entire state space, interpolating outside the grid and checking that the equilibrium conditions are satisfied at all grid points. A key feature of this algorithm is that the two financial constraints are allowed to bind only in some states of nature. The algorithm also captures the high non-linearities close to these constraints.

4.7 Calibration

We calibrate the model to an annual frequency using data from the U.S. economy from 1984.I-2010.II. Parameter values are summarized in Table 1.

Functional Forms— We make the following assumptions regarding functional forms for preferences and technology:

$$\begin{aligned}
u(c-G(n)) &= \frac{\left[c-\chi\frac{n^{1+\frac{1}{\omega}}}{1+\frac{1}{\omega}}\right]^{1-\sigma}-1}{1-\sigma} \\
F(z,k,h) &= zk^{\alpha}h^{1-\alpha} \\
\psi(k_t,k_{t+1}) &= \frac{\phi_k}{2}\left(\frac{k_{t+1}-k_t}{k_t}\right)^2 k_t
\end{aligned}$$

Stochastic Processes— We model the shocks to the interest rate and productivity as a first-order bivariate autoregressive process:

$$\begin{pmatrix} \hat{z}_t \\ \hat{R}_t \end{pmatrix} = \begin{pmatrix} \bar{z} \\ \bar{R} \end{pmatrix} + \rho \begin{pmatrix} \hat{z}_{t-1} \\ \hat{R}_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{R,t} \end{pmatrix}$$

where $\varepsilon_t = [\varepsilon_{z,t}\varepsilon_{R,t}]'$ follows a bivariate normal distribution with zero mean and contemporaneous variance-covariance matrix V. To construct these series, we use take the ex-post real interest rate on the 3 month US-Treasury Bills for the interest rate, and we follow the standard Solow residuals approach to construct the series for TFP. Our OLS estimation yields the following values:

$$\rho = \begin{bmatrix} 0.755972 & -0.030037 \\ -0.074327 & 0.743032 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} 0.0000580 & -0.0000107 \\ -0.0000107 & 0.0001439 \end{bmatrix}$$

Financial shocks are modeled as an independent process following a two-state Markov chain with values given by $\{\kappa^L, \kappa^H\}$ and transition matrix:

$$P = \begin{bmatrix} P_{L,L} & 1-P_{L,L} \\ 1-P_{H,H} & P_{H,H} \end{bmatrix}$$

Parameter Values— We need to assign values to 16 parameters that we classify in two sets. The first subset includes parameters that are chosen independently of equilibrium conditions or are calibrated using steady state targets, some of which are typical in the business cycle literature. This subset is given by $\{\alpha,\delta,\omega,\beta,R,\varphi,\chi\}$. The capital share α is set to 0.33; the depreciation rate is set at 10 percent; the risk aversion σ is set to 2; the Frisch elasticity of labor supply in the GHH preference specification ω is set to 2.[13] The value of β is pinned down by setting the capital-output ratio equal to 2.5 in a deterministic steady state with $\kappa = 0$, which results in a value of 0.964.[14] $\bar{R}$ is set so that the average interest rate is equal to 1.015 percent which is the average interest rate in the period considered.

The efficiency cost φ is more specific to our framework. For this parameter, we choose a benchmark value of 50 bps. Considering that financial intermediation represent about 5 percentage points of GDP, this implies that cost of public supply of credit is 10 percent higher than the private one [15] Finally, we normalize the labor disutility coefficient χ and the average value of the TFP shock so that employment and output equal one in the deterministic steady state.

The remaining six parameters are $\left\{\phi_k,\kappa^L,\kappa^H,\bar{d},P_{L,L}P_{H,H}\right\}$ and are set to jointly match a set of six long-run moments from the economy where bailouts are absent.[16] These moments are: (1) a standard deviation of investment of 10 percent; (2) an average leverage ratio of 45 percent; (3) four credit crunches occurring every 100 years; (4) an average duration of a credit crunch of 3 years; (5) a probability of a binding dividend constraint equal to the probability of a binding collateral constraint; and (6) a probability of a binding collateral constraint conditional on $\kappa = \kappa^H$ equal to zero. While all these parameters affect all the target moments, each parameter has a more significant impact on one particular moment, as we explain below.

[13] Notice that to make these preferences consistent with long-run growth, one needs to assume that technological progress increases the utility of leisure. As shown by Benhabib, Rogerson, and Wright (1991), these preferences can be interpreted as reduced form preferences for an economy with home production and technological progress in the home production sector.

[14] Due to precautionary savings, average capital is 2.6, which is still within the range of empirical estimations.

[15] We also note that given the rest of the parameter values, we find that financing the bailout with a distortionary tax on labor and setting $\varphi = 0$ yield very similar results that our benchmark value for φ.

[16] To the extent that optimal bailouts in our model have similar features to the data, one could argue that it would be more appropriate to target the moments of the economy with the optimal bailout policy. Since we will experiment with different values of φ,, it is more practical to calibrate the economy without bailouts. In any case, this choice does not have a significant impact on the results.

We set the value of κ^H high enough so that the collateral constraint never binds when κ takes this value. The value of κ^L is set to target an average leverage of 45 percent. The choice of a leverage ratio of 45 percent corresponds the ratio of credit market instruments to net worth in the years preceding the 2007 financial crisis (see Table B102 in the Flow of Funds database).

Table 1: Calibration

	Value	Basis
Mean Interest rate	$R-1=0.015$	3 month US-Treasury Bills
Discount factor	$\beta=0.97$	Capital-output= 2.5
Depreciation rate	$\delta=0.1$	Standard value
Share of capital	$\alpha=0.34$	Standard Value
Labor disutility coefficient	$\chi=0.66$	Normalization
Risk aversion	$\sigma=2$	Standard value
Frisch elasticity parameter	$\omega=2.0$	Benchmark value
Efficiency cost	$\varphi=50bps$	Benchmark value
Parameters set by simulation	Value	Target
Financial shock		
	$\kappa_L=0.43$	Average leverage =45 percent
	$\kappa_H=0.54$	Non-binding collateral constraint
	$P_{HH}=0.9$	Probability of credit crunch = 4 percent
	$P_{LL}=0.1$	Duration of credit crunch = 3 years
Adjustment cost	$\phi_k=2.2$	SD of investment = 9 percent
Dividend threshold	$\bar{d}=0.035$	Equalize prob. binding constraints

The dividend threshold $\bar{d}$ is more difficult to pin down from the data because it is difficult to identify whether constraints on equity financing or on borrowing are more pervasive. For simplicity, we set $\bar{d}$, so that the borrowing constraint and the dividend payout constraint bind with the same probability in the long run. This yields $\bar{d}=0.035$ and probabilities of binding constraints equal to 8 percent. The adjustment cost on capital is calibrated to match the standard deviation of investment between 1950-2010. This calibration yields $\phi_k=2.2$, a standard value.

We calibrate the transition matrix for the financial shock to target the frequency and the duration of financial crises. We define a financial crisis as an episode in which credit falls below two standard deviations. The financial crisis begins in the period in which credit falls below one standard deviation, providing that some point within the next two years, the level of credit falls at least two standard deviations below its mean. The crisis ends when the level of credit exceeds one standard deviation below its mean. Consistent with the empirical literature (e.g. Reinhart and Rogoff, 2009), we target an incidence of crises of 4 every 100 years and an average duration of 3 years. This procedure yields that $P_{L,L}$, which mostly affects the duration of crises, equals 0.1. $P_{H,H}$, which primarily affects the long-run probability of a crisis, equals 0.9. With these values, the economy spends only 11 percent of the time with negative financial shocks.

5 Results of the Quantitative Analysis

We begin by analyzing the behavior of the model in terms of both real aggregate variables and aggregate flows of financing. Then we analyze the workings of the model and the implications of bailouts on the real economy and welfare.

5.1 Crises Comparison

We now describe how the model performs relative to the data. We focus on two dimensions. First, we construct a crisis event analysis and compare it to the Great Recession in the US economy. Second, we compute both business cycle moments and financial moments and compare them with the counterparts in the US data from 1983:1-2010:2 (annualized). As in Jermann and Quadrini (2012), this period is chosen to focus on post-financial globalization period. (See their appendix for data construction).

Financial Crises:— We construct a crisis event for the economy when the optimal policy is implemented according to the following steps. First, we run a long time-series simulation after feeding a sequence of shocks, drawn from the distribution of the stochastic processes for (κ, R, z). Second, we identify the crisis episodes, defined as episodes in which credit falls by more than two standard deviations. This provides a distribution of crises episodes. To make the comparison with the Great Recession more comparable, we extract the episode where the fall in GDP following the crisis is close to 7.7 percent, which is the drop in GDP between 2008:2-2009:1, with TFP and the interest rate being slightly below trend, and we compute the mean of macro variables of all these episodes. The result of this exercise is reported in Table 2.

Table 2: Crises Comparison [a]

	No Intervention [b]	Optimal Policy [c]	Great Recession [d]
Output	-7.9	-7.4	-7.7
Hours	-4.9	-4.7	-9.80
Investment	-44.7	-38.8	-38.8
Consumption	-5.1	-6.2	-6.4
Debt-repurchase	12.0	11.0	7.1
Equity-Payouts	-0.7	-0.7	-1.000

[a] The table reports macro effects during financial crises for the economy with no intervention, the economy with optimal policy and for the US Great Recession. For the model simulations, all variables are expressed in terms of deviation with respect to the mean of the optimal bailout policy (output, hours and investment are percentage deviations)
[b] Magnitudes for the optimal policy correspond to the average macro responses conditional on an output drop of 7.7 percent following a financial crisis, a shock to productivity equal to the mean and an interest rate below the mean.
[c] Magnitudes for No Intervention correspond to a counterfactual exercise where no bailout takes place conditional on the same initial states as in the crises of the Optimal Policy (see note b).
[d] Data: output, hours, consumption and investment are deviations relate to HP trend.

Table 2 also shows a counterfactual policy experiment in which the government does not conduct bailouts. This experiment consists of taking the crisis episodes identified in the economy with the optimal bailout policy and ask what would be the response of the economy without bailout policy. That is, given the initial state X_t at the crisis identified above and the policy rules.[17]

The results in Table 2 shows that the model performs relatively well when compared to the Great Recession. On the real side, there is an important contraction in employment, consumption and investment. On the financial side, there is also significant deleveraging as well as a contraction in equity payouts.

The results of the counterfactual' experiment shows that without intervention the credit crunch is substantially more server. The fall in output and employment would is 20 percent higher than

[17]The counterfactual experiment assumes implicitly that what we observe in the data is the optimal policy. We could take a different stance but the significant differences with and without bailouts would remain.

without intervention. Finally, we note that the average optimal bailout that corresponds to the optimal policy within the event is 2 percentage points of GDP. It is important to note that this counterfactual experiment does not take into account explicitly the anticipation effects of bailouts, as both economies start with the same initial state variables. We will analyze these incentives below.

5.2 Optimal Intervention

We begin by showing the equilibrium policy functions of the model with and without policy. We then proceed to simulate the model and assess the macroeconomic and welfare implications of bailouts.

5.2.1 Laws of Motion

Figure 1 shows the laws of motion for debt, capital, and leverage in the economy as a function of the current level of debt, for a value of capital approximately equal to the average value and an average TFP shock. Since mean output is approximately one, all variables can be interpreted as ratios with respect to the average output. The superior (inferior) panel corresponds to a positive (adverse) financial shock. The straight lines correspond to the competitive equilibrium without bailouts (NBP), and the dashed lines correspond to the economy with the optimal bailout policy (OBP). Let us first describe the behavior of NBP before analyzing the effects of credit intervention.

As Figure 1 shows, the occasionally binding constraints produce a non-monotonic law of motion for debt. For low values of current debt, the collateral constraint is not binding. As the value of current debt increases, the demand for debt increases. When the current level of debt reaches 1.2, the collateral constraint becomes binding. Since firms need to cut down on investment, the borrowing capacity shrinks. As a result, for $b > 1.2$ in the inferior panel, the next period debt holdings decrease in current debt holdings. For this law of motion, the dividend payout constraint becomes binding approximately at the same value of debt.

In order to understand how bailouts affect the competitive equilibrium, it is useful to first analyze its effects during periods in which the financial constraint becomes binding. As Figure 1 shows, bailouts allow firms to borrow more during these periods because as firms receive the transfers, they can allocate these funds to invest more in capital, which boosts their capacity to borrow.

In the region where the financial constraints are not binding, firms also borrow more in the competitive equilibrium with bailouts because there is a lower incentive to accumulate precautionary savings during normal times since crises become less severe as a result of bailouts. Higher borrowing will lead in turn to a higher probability that the economy becomes financially constrained in the future. This effect is markedly stronger when the economy is in a positive financial shock and has a relatively large amount of debt.

As Figure 1 shows, OBP features more capital accumulation when financial constraints become binding because bailouts provide more funds, which allows firms to invest more. The effects before the constraint bind are generally ambiguous due to two opposing forces. On one hand, the fact that OBP effectively has more insurance available increases the demand for risky assets, in this case, capital. On the other hand, since capital is also a form of savings, the demand for capital decreases. Overall, we find that there is less demand for capital in OBP, except for relatively low values of the capital stock.

5.3 Non-linear Impulse Responses

This section conducts simulations of the economy with and without bailouts to demonstrate how the time-series are affected by the presence of bailouts.

Our first experiment is a non-linear impulse response. In particular, we simulate the economy with a constant TFP equal to the average and a financial shock that has a high value at the beginning of the simulations, then falls in what we refer to as the credit crunch, and then again achieves a high value. For illustrative purposes, we start the economy at a level of bonds and capital at which the competitive equilibrium converges after a long sequence of κ^H and a TFP shock equal to the average. In this experiment, the shock hits after 40 periods (years) so that the economy with the optimal bailout remains approximately constant in the absence of any other shock. We also compute the simulations of the economy where government intervention is unanticipated when the negative financial shock hits. After the initial "surprise," we assume for simplicity that the economy returns in the following period to a situation in which there is a policy of no bailouts.

That is, the government switches from NBP to OBP in period 40 and then returns to NBP the following period. Each of these switches is unanticipated.

Figures 2 and 3 show the simulations for the economy with the optimal bailout policy (OBP), for the economy without bailouts (NBP), and for the economy where the bailout is unanticipated (UBP). The variables we consider in Figure 2 are credit, leverage, output, employment, investment, and the exogenous shocks. Figure 3 plots the sequence of dividends, risk premium, bailouts, and taxes on debt as defined in Proposition 2.

When the bailout is unanticipated, the economy does not experience as sharp a decline in credit flows, investment, and output, as shown in Figure 1. Instead, when the bailout policy is anticipated, there is a trade-off: on one hand, the economy increases the amount of debt in response to the insurance provided by the government's intervention; on the other hand, the bailout relaxes balance sheet constraints ex-post. In fact, as the negative financial shock hits in period 40, both economies experience sharp deleveraging effects. For the optimal bailout policy, however, the effects of the credit crunch are less protracted. In fact, it takes 2 more years to recover approximately 75 percent of the output loss when there is no government intervention. Moreover, the cumulative loss in output for NBP is 11.2 percent, whereas it is 6.6 percent for OBP.

Figure 3 also shows that the magnitude of the bailout in the simulated credit crunch is approximately two percentage points of GDP. Considering that the cumulative loss in output is reduced by 4 percent, the multiplier effect of the bailout over output is approximately 2.2. Finally, figure 3 also shows the impact of bailouts on risk premium measured as the difference between the expected return on capital for firms and the return on bonds. Interestingly, at t=0, risk premium is smaller for the economy with bailouts. However, as leverage increases over the experiment, risk premium increases slightly for the economy for OBP.

We now examine the importance of the prudential instruments to complement the bailout policy. As Figure 3 shows, OBP features a tax on debt during the tranquil times preceding the credit crunch. The tax is initially around 0.12 percent and rises to 0.2 percent as the amount of leverage increases in the run up to the crisis.

5.4 Moral Hazard

We now conduct a series of experiments to gauge the importance of the moral hazard effects. To be sure, the increase in leverage and risk-taking in the previous section is efficient in the sense that this is the optimal response to the higher level of insurance provided by bailouts, and the planner offsets the incentive to overborrow by taking debt. That is, during the optimal intervention, the cost of borrowing is still the socially optimal one because the planner can control the level of borrowing. To see the importance of the prudential policy, we consider the two cases described above: systemic bailout and idiosyncratic bailout policy.

We reconstruct the non-linear impulse response from Figure 1, but now we compare the economy without bailouts (NBP) with the economy with bailouts but without a prudential tax on debt. Figure 4 shows the results of this experiment.

As Figure 4 shows, the lack of prudential policy results in an even larger amount debt relative to the economy where prudential policy is used together with a bailout policy. This increase in debt leads to a sudden crash in period 40, which generates a recession that is now comparable to the economy without credit intervention. In fact, the cumulative loss in output is now about the same for the economy with bailouts and for the economy without bailouts. Moreover, the economy without prudential policy remains exposed to more frequent credit crunches.

5.5 Welfare

Next, we compute the welfare gains from policy intervention. We consider the three cases analyzed above: (a) optimal intervention; (b) systemic bailout without prudential policy; (c) idiosyncratic bailout without prudential policy. We compute for every possible state the percentage increase in consumption that leaves a household indifferent between living in an economy with the optimal government policy and remaining in an economy without government intervention. The results from these calculations are plotted in Figure 6 for an average TFP shock and an adverse financial shock. Note that the welfare gains are positive in all states and reach the maximum levels when the economy is highly leveraged for the optimal policy.[18] Overall, the welfare gains from implementing OBP are on average close to 0.1 percentage points of permanent consumption and

[18]For graphical purposes, we attach a value of zero for those states that are non feasible, i.e. those with very high levels of debt.

are comparable to the gains obtained in the macro-prudential policy literature based on systemic risk externalities.[19]

The case of systemic bailout policy delivers welfare gains which are quite close to the optimal policy. On the other hand, idiosyncratic bailouts deliver substantial welfare losses.

6 Conclusion

We developed a quantitative framework to examine the effects of bailouts. The key novelty of our work is that we investigate what is the optimal bailout policy considering both the effects of relaxing balance sheet constraints ex-post and the increase in leverage that occurs ex-ante.

Overall, we find that the optimal intervention requires a bailout of approximately two percentage points of GDP during a credit crunch. While this policy slightly increases the exposure to a credit crunch, it generates larger gains by significantly reducing the effects of a credit crunch on the real sector. In particular, the loss in output falls by more than 40 percent when the government implements the optimal bailout policy.

Our findings are particularly relevant to the ongoing debates about the appropriate role of central banks during financial crises. Our analysis suggests that systemic insurance provided by the government should be part of the financial system's safety net. The design of this systemic insurance, however, should be complemented by macro-prudential policy during good times to offset incentives for excessive risk-taking.

One qualification of our analysis is that we do not allow bailouts to affect the degree of market incompleteness in the economy. It is possible that government bailouts affect the incentives of the financial sector to supply hedging instruments. To the extent that there are restrictions to supply these instruments, our analysis suggests more broadly that the government should support the development of these markets of private insurance.

[19]However, maximum welfare gains across the ergodic distribution are larger here because bailouts have a large current impact on welfare. On the other hand, the welfare gains from prudential policy peak during good times because they reduce expected future volatility (see Bianchi and Mendoza (2010), Bianchi (2011))

References

BENHABIB, J., R. ROGERSON, AND R. WRIGHT (1991): "Homework in macroeconomics: Household production and aggregate fluctuations," *Journal of Political economy*, pp. 1166–1187.

BENIGNO, G., H. CHEN, C. OTROK, A. REBUCCI, AND E. YOUNG (2012): "Optimal Policy for Macro-Financial Stability," Mimeo, University of Virginia.

BERNANKE, B., AND M. GERTLER (1989): "Agency costs, net worth, and business fluctuations," *American Economic Review*, (1), 14–31.

BIANCHI, J. (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101(7), 3400–3426.

BIANCHI, J., AND E. G. MENDOZA (2010): "Overborrowing, Financial Crises and 'Macroprudential' Policy," NBER Working Paper No. 16091.

BIGIO, S. (2010): "Endogenous liquidity and the business cycle," *Mimeo, Columbia University*.

——— (2011): "Financial Risk Capacity," Mimeo, NYU.

CABALLERO, R., AND A. KRISHNAMURTHY (2001): "International and domestic collateral constraints in a model of emerging market crises," *Journal of Monetary Economics*, 48(3), 513–548.

CHARI, V., AND P. KEHOE (2009): "Bailouts, time inconsistency and optimal regulation," Federal Reserve Bank of Minneapolis Staff Report, November.

DEL NEGRO, M., G. EGGERTSSON, A. FERRERO, AND N. KIYOTAKI (2010): "The great escape? A quantitative evaluation of the FedŠs non-standard policies," Mimeo, Federal Reserve Bank of New York.

DIAMOND, D., AND R. RAJAN (2009): "Illiquid Banks, Financial Stability, and Interest Rate Policy Douglas," Forthcoming Journal of Political Economy.

DURDU, B., AND E. G. MENDOZA (2006): "Are asset price guarantees useful for preventing Sudden Stops?: A quantitative investigation of the globalization hazard–moral hazard tradeoff," *Journal of International Economics*, 69(1), 84–119.

FARHI, E., AND J. TIROLE (2010): "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," Forthcoming, American Economic Review.

GERTLER, M., AND P. KARADI (2011): "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58(1), 17–34.

GERTLER, M., AND N. KIYOTAKI (2010): "Financial intermediation and credit policy in business cycle analysis," Handbook of Monetary Economics, forthcoming.

GERTLER, M., N. KIYOTAKI, AND A. QUERALTO (2011): "Financial Crises, Bank Risk Exposure and Government Financial Policy," Mimeo, NYU.

GILCHRIST, S., J. SIM, AND E. ZAKRAJŠEK (2010): "Uncertainty, Financial Frictions, and Investment Dynamics," .

GUERRIERI, V., AND G. LORENZONI (2011): "Credit Crises, Precautionary Savings and the Liquidity Trap!," University of Chicago mimeo.

HE, Z., AND A. KRISHNAMURTHY (2012): "A Macroeconomic Framework for Quantifying Systemic Risk," Mimeo, University of Chicago.

HOLMSTRÖM, B., AND J. TIRÓLE (1998): "Private and Public Supply of Liquidity," *The Journal of Political Economy*, 106(1), 1–40.

JEANNE, O., AND A. KORINEK (2010): "Managing Credit Booms and Busts: A Pigouvian Taxation Approach," NBER Working Paper.

——— (2011): "Macroprudential Regulation Versus Mopping Up After the Crash," Mimeo, University of Maryland.

JERMANN, U., AND V. QUADRINI (2012): "Macroeconomic effects of financial shocks," *American Economic Review*, 102(1), 238–271.

KEISTER, T. (2010): "Bailouts and financial fragility," Mimeo, Federal Reserve Bank of New York.

KEISTER, T., AND V. NARASIMAN (2011): "Expectations versus fundamentals: does the cause of banking panics matter for prudential policy?," note=Mimeo, Federal Reserve Bank of New York.

KELLY, B., H. LUSTIG, AND S. VAN NIEUWERBURGH (2011): "Too-Systemic-To-Fail: What Option Markets Imply About Sector-wide Government Guarantees," NBER Working Paper.

KIYOTAKI, N., AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

LORENZONI, G. (2008): "Inefficient Credit Booms," *Review of Economic Studies*, 75, 809–833.

MENDOZA, E. G. (2010): "Sudden Stops, Financial Crises, and Leverage," *The American Economic Review*, 100(5), 1941–1966.

MIDRIGIN, V., AND T. PHILIPPON (2011): "Household Leverage and the Recession," CEPR Discussion Paper No. DP8381.

MILLER, M., AND F. MODIGLIANI (1961): "Dividend policy, growth, and the valuation of shares," *the Journal of Business*, 34(4), 411–433.

MYERS, S., AND N. MAJLUF (1984): "Corporate financing and investment decisions when firms have information that investors do not have," *Journal of financial economics*, 13(2), 187–221.

PERRI, F., AND V. QUADRINI (2011): "International recessions," NBER Working Paper.

REINHART, C., AND K. ROGOFF (2009): *This time is different: Eight centuries of financial folly*. Princeton Univ Pr.

SCHNEIDER, M., AND A. TORNELL (2004): "Balance sheet effects, bailout guarantees and financial crises," *Review of Economic Studies*, 71(3), 883–913.

SHLEIFER, A., AND R. VISHNY (1997): "A Survey of Corporate Governance," *The Journal of Finance*, 52(2), 737–783.

SHOURIDEH, A., AND A. ZETLIN-JONES (2012): "External financing and the role of financial frictions over the business cycle: Measurement and theory," Mimeo, Wharton.

WOODFORD, M. (1990): "Public debt as private liquidity," *The American Economic Review*, pp. 382–388.

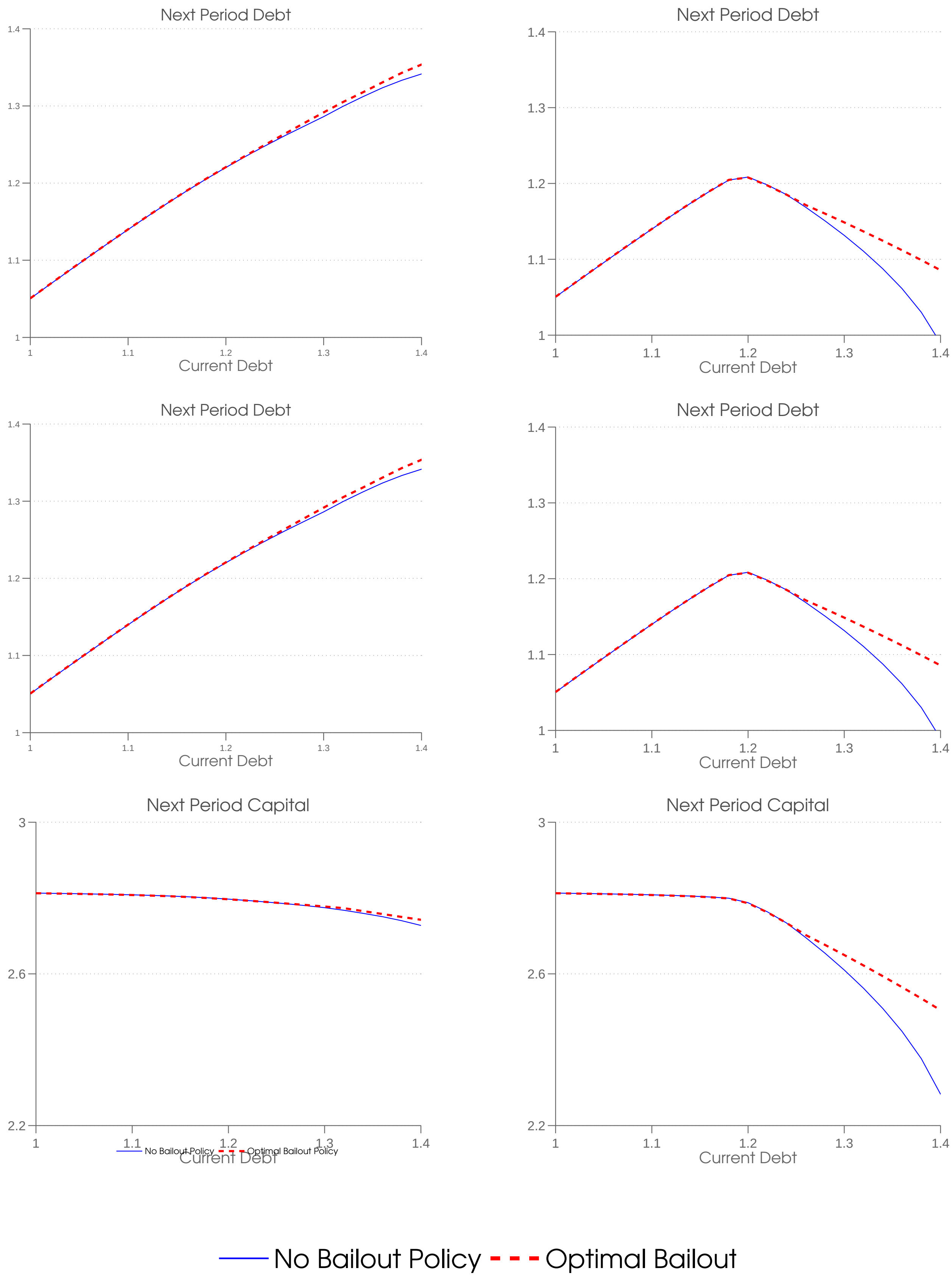


Figure 1: Laws of Motion for average value of capital and average shocks. The left (right) column corresponds to κ^H (κ^L)

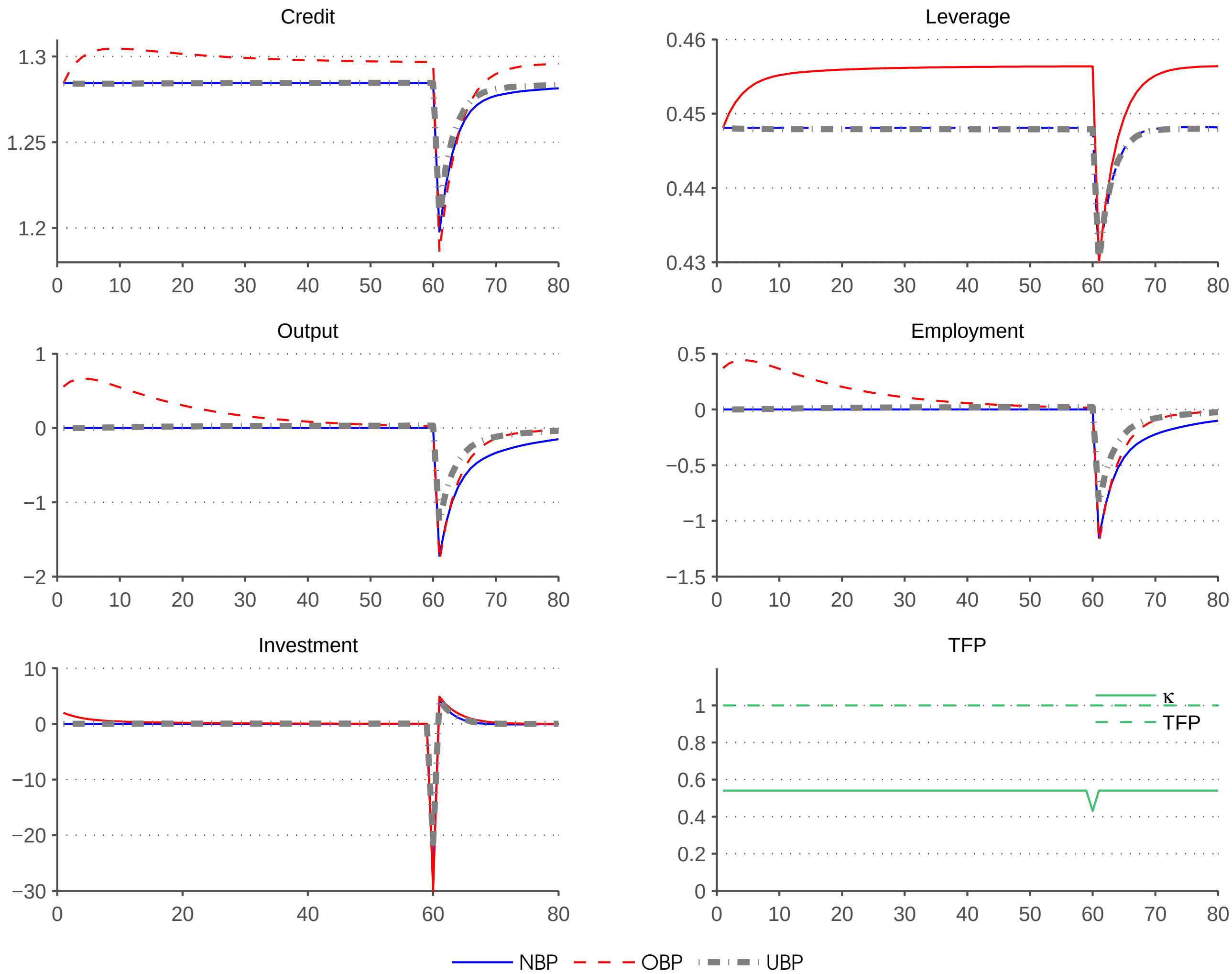


Figure 2: Non-linear Impulse Responses

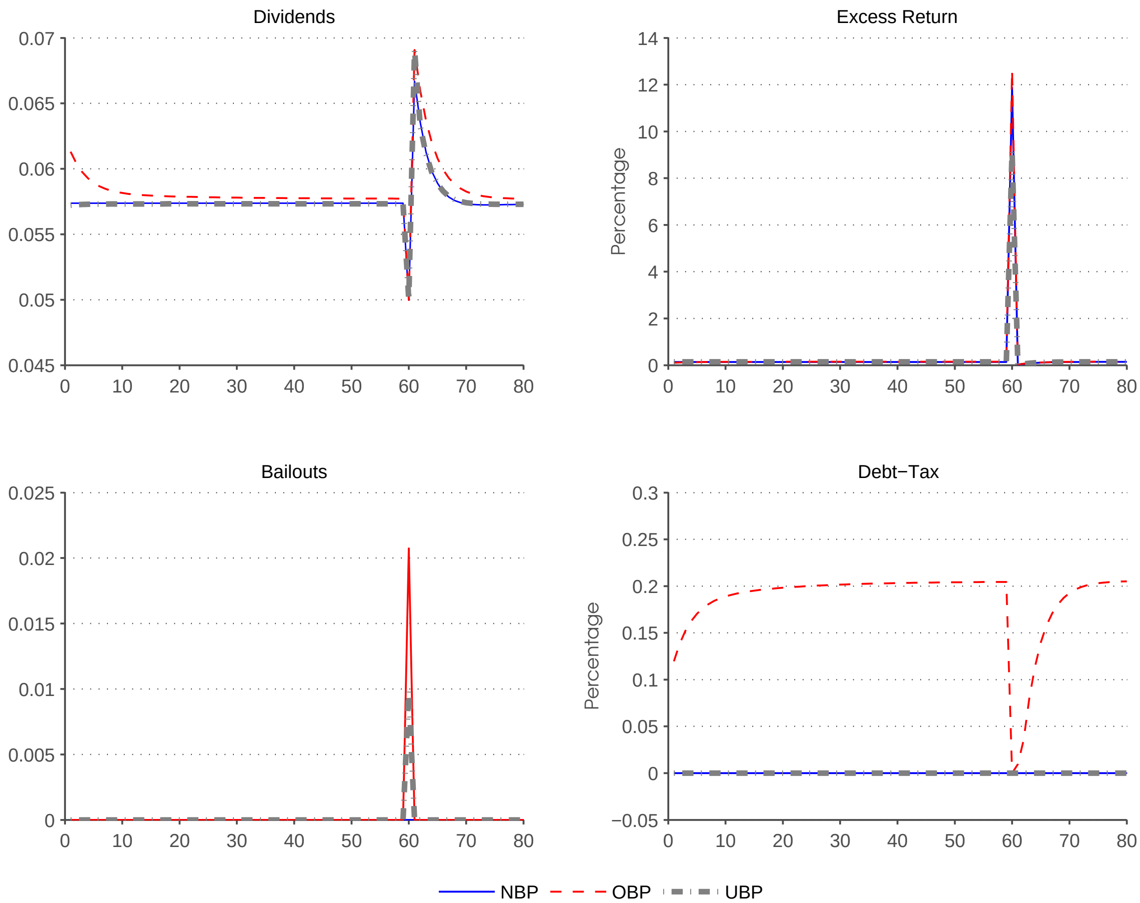
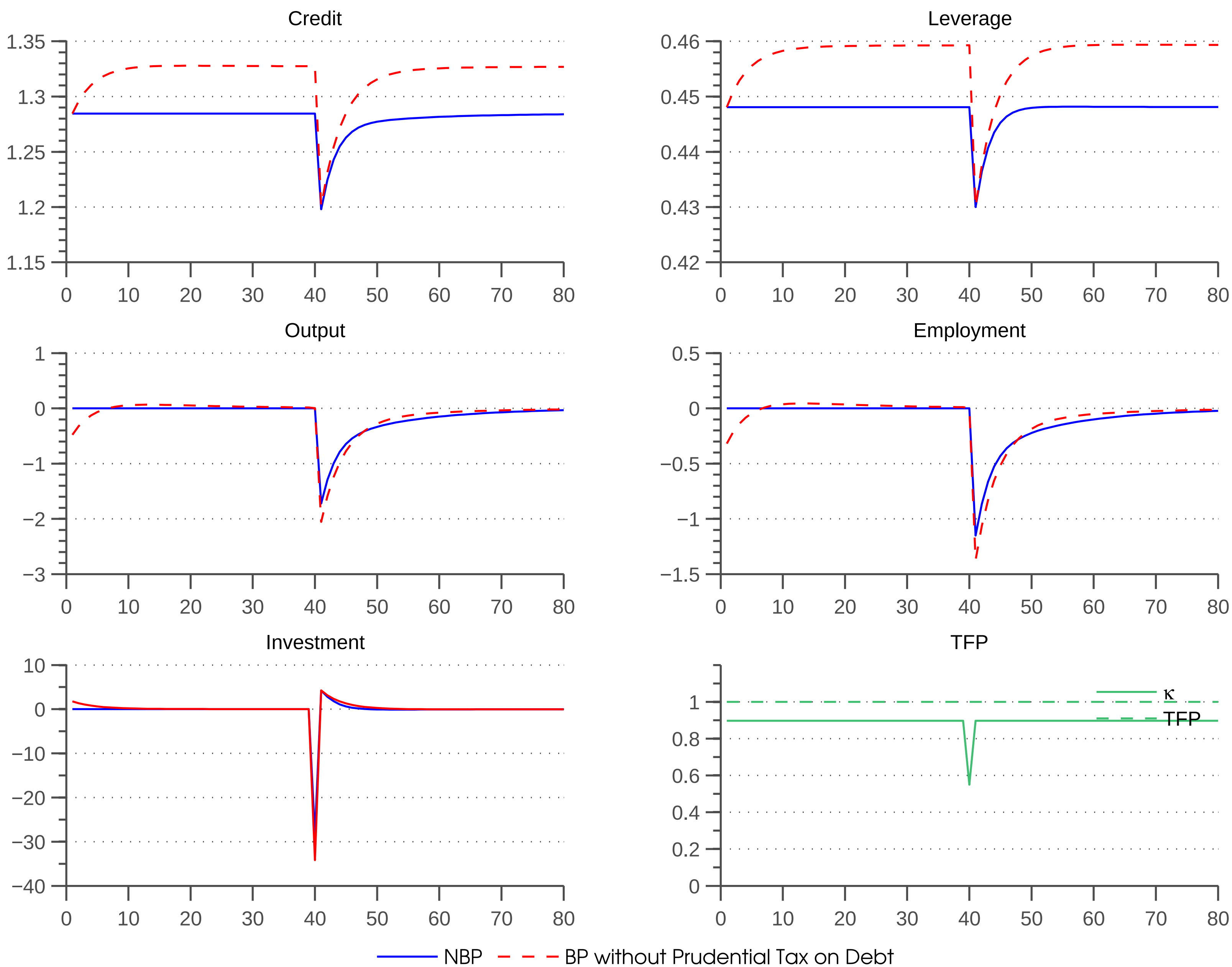


Figure 3: Non-linear Impulse Responses

Figure 4: Non-linear Impulse Responses without Prudential Policy



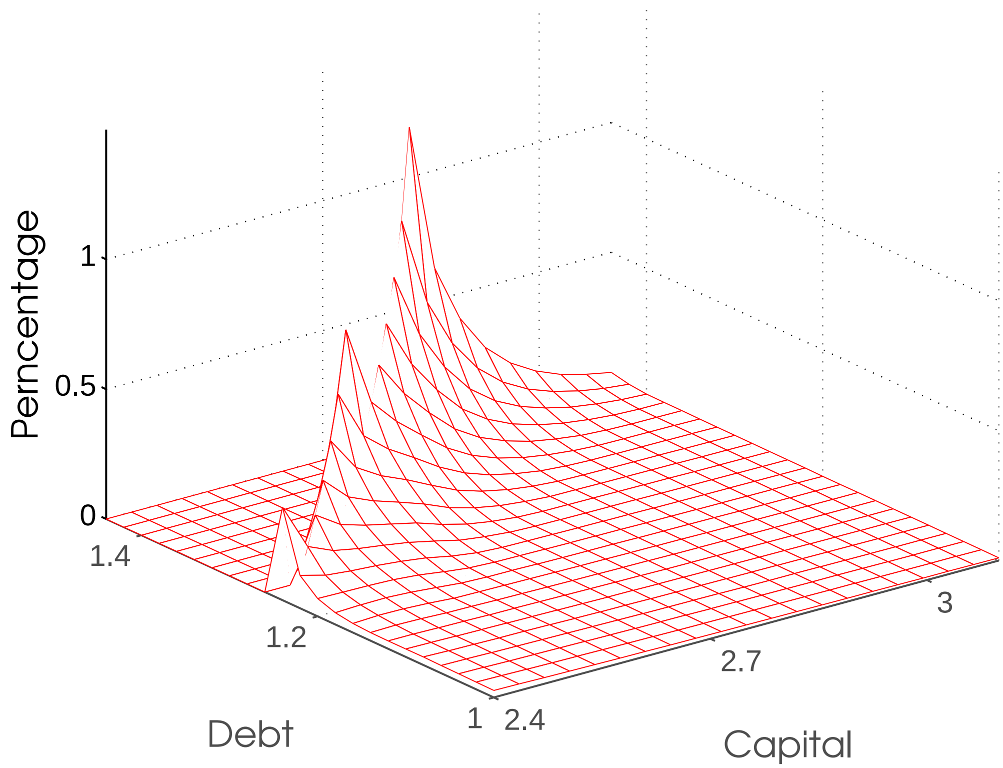


Figure 5: Welfare Gains from optimal policy for $\kappa = \kappa^L$ and average TFP shock

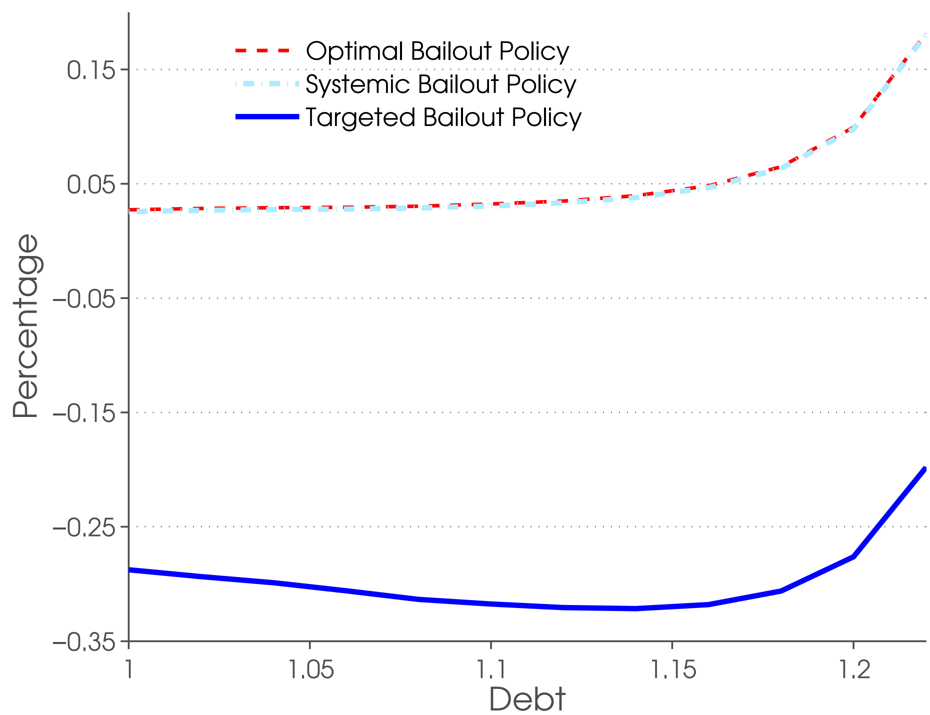


Figure 6: Welfare Gains for different policies for an initial value of capital equal to the mean in the simulation, $\kappa = \kappa^L$ and an average TFP shock

7 Appendix

Decentralization based of planner's problem with equity injections. An alternative policy to the debt relief policy analyzed in the text is the injection of equity. In order to facilitate a comparison with debt relief, we consider an equity injection that is a fraction of the amount of individual debt held by the firm, i.e., the number of new shares issued is such that the value of equity injections equals $e_t b_t$. In particular, the government mandates firms to issue new shares and transfer those shares to the households, which then receive the future dividend payments.

The firms' objective can be expressed as maximize $\mathbb{E}_t \sum_{j=0}^{\infty} m_{t+j}(d_{t+j} - e_t b_t)$ subject to the flow of funds constraint $b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq F(z_t, k_t, h_t) - w_t n_t + \frac{b_{t+1}}{R_t} + e_t b_t(1-\tau_t) + T_t^f$ and the same financial constraints in problem (10). The first-order condition with respect to debt yields:

$$1 + \eta_t = R_t \mathbb{E}_t m_{t+1}(1 + \eta_{t+1}(1 - e_{t+1})) + R_t \mu_t \tag{24}$$

At the beginning of each period, the total number of shares can be renormalized to one. Hence, the rest of the equilibrium conditions remain the same yielding a similar proposition to the one stated for debt relief.

Proposition 2 *The government can implement the constrained optimal allocations through a combination of equity injections, taxes on debt and capital, and lump sum taxes. In particular:*

$$e_t = \frac{\Upsilon_t}{b_t}, \quad T_t = \Upsilon_t(1+\varphi), \quad \tau_t^b = \frac{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}(1-e_{t+1})) + \mu_t} - 1, \; T_t^f = \frac{b_{t+1}}{R_t}\tau_t \tag{25}$$

where all variables are evaluated at the constrained optimal allocations.

The proof follows the same steps as Proposition 3. Note from (24) that the shadow cost of tightening the dividend payout constraint in a state $t+1$, in which the government is recapitalizing firms, is reduced by a factor of $(1 - e_{t+1})$. Comparing this condition with (23) yields that the tax on debt is smaller than the one required for debt relief. Intuitively, equity injections also involve a cost for shareholders because they perceive a reduction in their ownership of the firm. In addition, the tax on debt is strictly positive only if $\varphi > 0$. Intuitively, firms do not internalize that when they are bailed-out, the social costs of the bailout are paid by other agents in the economy. As a result, they would take too much debt in the absence of a tax on debt.